CS 280: Homework 4

Date of Handout: 20 Oct 99
Due Date: 27 Oct 99 in class
The alphabet is $\{0, 1\}$ in Problems 1, 2 and 4.
Problem 1: Write NFAs to recognize the following languages:

(a) *(5 pts)* The set of all strings having a 00 which is followed by a 11.

(b) *(5 pts)* The set of all strings xy such that if m and n are the number of 1s in x and y, respectively, then $(m - n)$ is divisable by 3.

Problem 2: Write regular expressions for the following languages:

(a) *(5 pts)* The set of all strings not containing 000.

(b) *(10 pts)* The set of all strings with an equal number of 0s and 1s such that no prefix has two more 0s than 1s or two more 1s than 0s. Begin by arguing that every prefix of even length of a string in this language must have an equal number of 0s and 1s.

Problem 3: Let the language L be any subset of Σ^*, where Σ is a finite alphabet. The language L induces the following relation on strings x, y in Σ^*:

> xRy if and only if for all strings $s \in \Sigma^*$, $xs \in L$ if and only if $ys \in L$.

(a) *(5 pts)* Prove that the relation R is reflexive, symmetric and transitive. Conclude that R is an equivalence relation.

(b) *(10 pts)* If L is accepted by a DFA, prove that R can have only finitely many equivalence classes.

Problem 4: *(optional) (10 pts)* Prove that the language

$$L = \{xy \Big| \, |x| = |y|, \text{ and the number 1s in } x \text{ and the number of 1s in } y \text{ are both even}\}$$

is not regular