CMSC 23700	**Introduction to Computer Graphics**	**Handout 2**
Fall 2003		**November 5**

A triangle mesh for an icosahedron

This handout provides describes the triangle mesh used to represent a icosahedron. An icosahedron has 20 faces, which are equilateral triangles, 30 edges, and 12 vertices. It is an interesting polyhedron, because it can serve as the starting point for sphere tessellation.

Consider an icosahedron is centered at the origin with the distance to each vertex being one. Using the principles of trigonometry, we can compute the locations of the vertices, but that is a lot of work. Fortunately, others have done for us. The following mesh is based on the treatment in *Geometric Tools for Computer Graphics* (Morgan Kaufmann 2003).

Let $t = \frac{(1+\sqrt{5})}{2}$, then the vertices of the icosahedron are

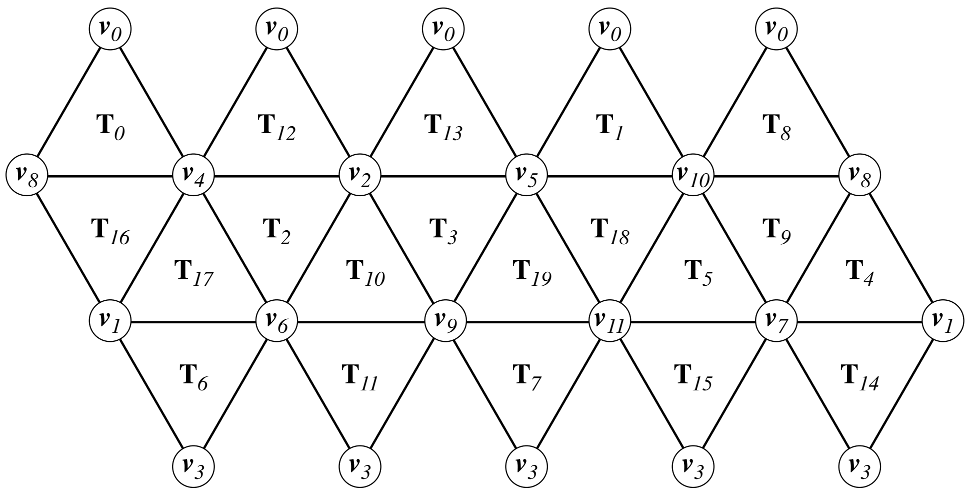
$$
\begin{array}{rclcrcl}
\mathbf{v}_0 & = & \frac{1}{\sqrt{1+t^2}}\langle t, 1, 0\rangle & \qquad & \mathbf{v}_6 & = & \frac{1}{\sqrt{1+t^2}}\langle -1, 0, t\rangle \\
\mathbf{v}_1 & = & \frac{1}{\sqrt{1+t^2}}\langle -t, 1, 0\rangle & & \mathbf{v}_7 & = & \frac{1}{\sqrt{1+t^2}}\langle -1, 0, -t\rangle \\
\mathbf{v}_2 & = & \frac{1}{\sqrt{1+t^2}}\langle t, -1, 0\rangle & & \mathbf{v}_8 & = & \frac{1}{\sqrt{1+t^2}}\langle 0, t, 1\rangle \\
\mathbf{v}_3 & = & \frac{1}{\sqrt{1+t^2}}\langle -t, -1, 0\rangle & & \mathbf{v}_9 & = & \frac{1}{\sqrt{1+t^2}}\langle 0, -t, 1\rangle \\
\mathbf{v}_4 & = & \frac{1}{\sqrt{1+t^2}}\langle 1, 0, t\rangle & & \mathbf{v}_{10} & = & \frac{1}{\sqrt{1+t^2}}\langle 0, t, -1\rangle \\
\mathbf{v}_5 & = & \frac{1}{\sqrt{1+t^2}}\langle 1, 0, -t\rangle & & \mathbf{v}_{11} & = & \frac{1}{\sqrt{1+t^2}}\langle 0, -t, -1\rangle
\end{array}
$$

The triangles are as follows (vertices are listed in counter-clockwise order):

$$
\begin{array}{rclrcl}
\mathrm{T}_0 & = & \langle \mathbf{v}_0, \mathbf{v}_8, \mathbf{v}_4\rangle & \mathrm{T}_{10} & = & \langle \mathbf{v}_2, \mathbf{v}_9, \mathbf{v}_{11}\rangle \\
\mathrm{T}_1 & = & \langle \mathbf{v}_0, \mathbf{v}_5, \mathbf{v}_{10}\rangle & \mathrm{T}_{11} & = & \langle \mathbf{v}_3, \mathbf{v}_9, \mathbf{v}_{11}\rangle \\
\mathrm{T}_2 & = & \langle \mathbf{v}_2, \mathbf{v}_4, \mathbf{v}_9\rangle & \mathrm{T}_{12} & = & \langle \mathbf{v}_4, \mathbf{v}_2, \mathbf{v}_0\rangle \\
\mathrm{T}_3 & = & \langle \mathbf{v}_2, \mathbf{v}_{11}, \mathbf{v}_5\rangle & \mathrm{T}_{13} & = & \langle \mathbf{v}_5, \mathbf{v}_0, \mathbf{v}_2\rangle \\
\mathrm{T}_4 & = & \langle \mathbf{v}_1, \mathbf{v}_6, \mathbf{v}_8\rangle & \mathrm{T}_{14} & = & \langle \mathbf{v}_6, \mathbf{v}_1, \mathbf{v}_3\rangle \\
\mathrm{T}_5 & = & \langle \mathbf{v}_1, \mathbf{v}_{10}, \mathbf{v}_7\rangle & \mathrm{T}_{15} & = & \langle \mathbf{v}_7, \mathbf{v}_3, \mathbf{v}_1\rangle \\
\mathrm{T}_6 & = & \langle \mathbf{v}_3, \mathbf{v}_9, \mathbf{v}_6\rangle & \mathrm{T}_{16} & = & \langle \mathbf{v}_8, \mathbf{v}_6, \mathbf{v}_4\rangle
\end{array}
$$

$$\begin{aligned} \mathrm{T}_7 &= \langle \mathbf{v}_3, \mathbf{v}_7, \mathbf{v}_{11} \rangle & \mathrm{T}_{17} &= \langle \mathbf{v}_9, \mathbf{v}_4, \mathbf{v}_6 \rangle \\ \mathrm{T}_8 &= \langle \mathbf{v}_0, \mathbf{v}_{10}, \mathbf{v}_8 \rangle & \mathrm{T}_{18} &= \langle \mathbf{v}_{10}, \mathbf{v}_5, \mathbf{v}_7 \rangle \\ \mathrm{T}_9 &= \langle \mathbf{v}_1, \mathbf{v}_8, \mathbf{v}_{10} \rangle & \mathrm{T}_{19} &= \langle \mathbf{v}_{11}, \mathbf{v}_7, \mathbf{v}_5 \rangle \end{aligned}$$

Graphically, the mesh has the following layout:



Note that some vertices (*e.g.*, $\mathbf{v}_0$ appear multiple times in this figure.

In C, we can represent this mesh as a table of 12 vertices, and a table of 20 triangles represented by 3 indices, plus a normal vector for a total of 66 words. We might also want to store edge information, which would take additional storage.