Entropy &
Hidden Markov Models

Natural Language Processing

CMSC 35100

April 22, 2003

Agenda

- Evaluating N-gram models
  - Entropy & perplexity
    - Cross-entropy, English
- Speech Recognition
  - Hidden Markov Models
    - Uncertain observations
    - Recognition: Viterbi, Stack/A*
    - Training the model: Baum-Welch

Evaluating n-gram models

- Entropy & Perplexity
  - Information theoretic measures
  - Measures information in grammar or fit to data
  - Conceptually, lower bound on # bits to encode
- Entropy: H(X): X is a random var, *p:* prob fn

$$H(X) = -\sum_{x \in X} p(x) \log_2 p(x)$$

  - E.g. 8 things: number as code => 3 bits/trans
  - Alt. short code if high prob; longer if lower
    - Can reduce
- Perplexity: 2^H
  - Weighted average of number of choices

Entropy of a Sequence

- Basic sequence

$$\frac{1}{n} H(W_1^n) = -\frac{1}{n} \sum_{W_1^n \in L} p(W_1^n) \log_2 p(W_1^n)$$

- Entropy of language: infinite lengths
  - Assume stationary & ergodic

$$H(L) = \lim_{n \to \infty} -\frac{1}{n} \sum_{W \in L} p(w_1, ..., w_n) \log p(w_1, ..., w_n)$$

$$H(L) = \lim_{n \to \infty} -\frac{1}{n} \log p(w_1, ..., w_n)$$

Cross-Entropy

- Comparing models
  - Actual distribution unknown
  - Use simplified model to estimate
    - Closer match will have lower cross-entropy

$$H(p,m) = \lim_{n\to\infty} -\frac{1}{n}\sum_{W\in L} p(w_1,...,w_n)\log m(w_1,...,w_n)$$

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Entropy of English

- Shannon’s experiment
  - Subjects guess strings of letters, count guesses
  - Entropy of guess seq = Entropy of letter seq
  - 1.3 bits; Restricted text
- Build stochastic model on text & compute
  - Brown computed trigram model on varied corpus
  - Compute (pre-char) entropy of model
  - 1.75 bits

Speech Recognition

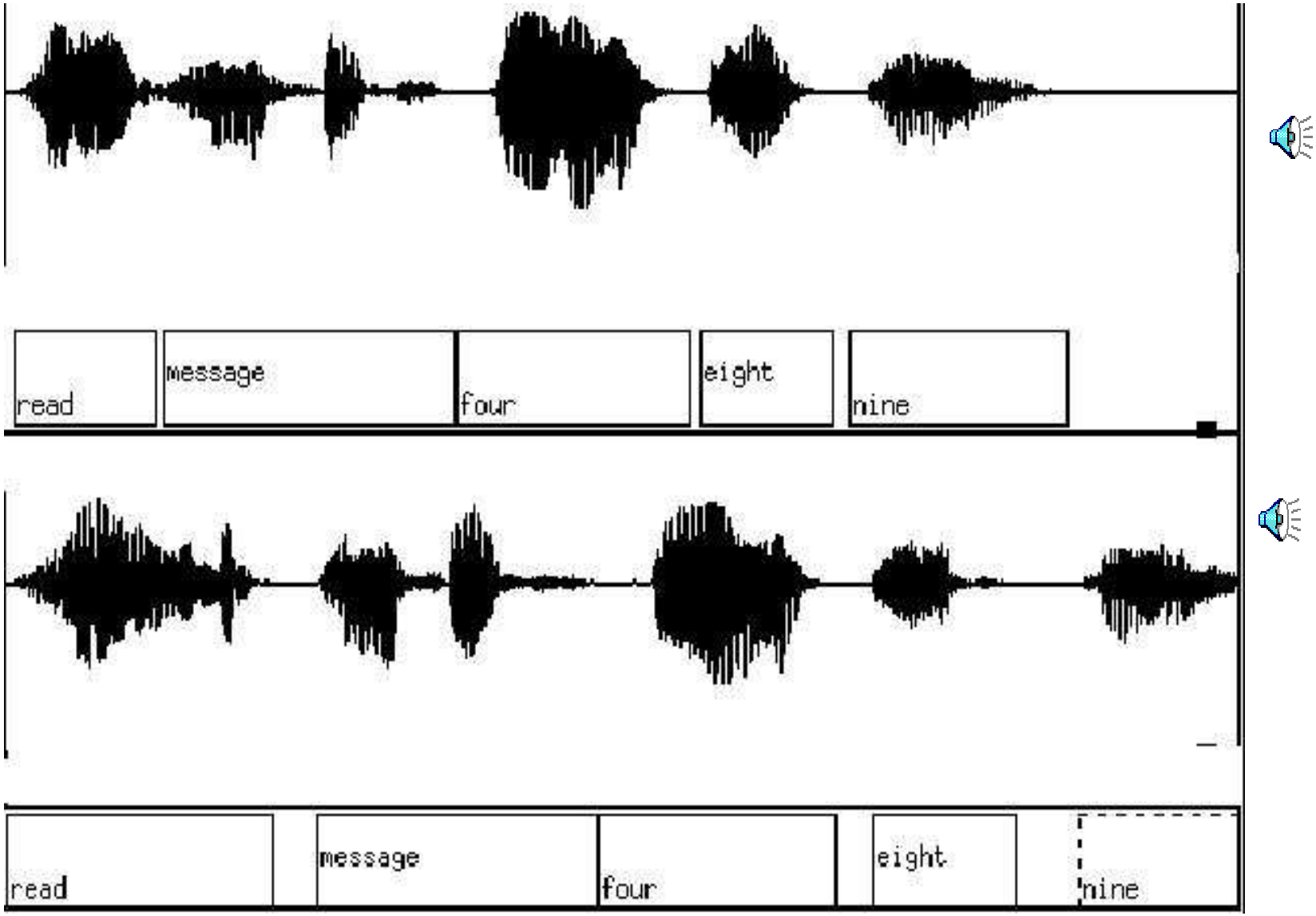
- Goal:
  - Given an acoustic signal, identify the sequence of words that produced it
  - Speech understanding goal:
    - Given an acoustic signal, identify the meaning intended by the speaker
- Issues:
  - Ambiguity: many possible pronunciations,
  - Uncertainty: what signal, what word/sense produced this sound sequence

Decomposing Speech Recognition

- Q1: What speech sounds were uttered?
  - Human languages: 40-50 phones
    - Basic sound units: b, m, k, ax, ey, …(arpabet)
    - Distinctions categorical to speakers
      - Acoustically continuous
    - Part of knowledge of language
      - Build per-language inventory
      - Could we learn these?

Decomposing Speech Recognition

- Q2: What words produced these sounds?
  - Look up sound sequences in dictionary
  - Problem 1: Homophones
    - Two words, same sounds: too, two
  - Problem 2: Segmentation
    - No “space” between words in continuous speech
    - “I scream”/”ice cream”, “Wreck a nice beach”/”Recognize speech”
- Q3: What meaning produced these words?
  - NLP (But that’s not all!)


read
message
four
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read
message
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Signal Processing

- Goal: Convert impulses from microphone into a representation that
  - is compact
  - encodes features relevant for speech recognition
- Compactness: Step 1
  - Sampling rate: how often look at data
    - 8KHz, 16KHz,(44.1KHz= CD quality)
  - Quantization factor: how much precision
    - 8-bit, 16-bit (encoding: u-law, linear…)

(A Little More) Signal Processing

- Compactness & Feature identification
  - Capture mid-length speech phenomena
    - Typically “frames” of 10ms (80 samples)
      - Overlapping
  - Vector of features: e.g. energy at some frequency
  - Vector quantization:
    - n-feature vectors: n-dimension space
      - Divide into m regions (e.g. 256)
      - All vectors in region get same label - e.g. C256

Speech Recognition Model

- Question: Given signal, what words?
- Problem: uncertainty
  - Capture of sound by microphone, how phones produce sounds, which words make phones, etc
- Solution: Probabilistic model
  - P(words|signal) =
  - P(signal|words)P(words)/P(signal)
  - Idea: Maximize P(signal|words)*P(words)
    - P(signal|words): acoustic model; P(words): lang model

Probabilistic Reasoning over Time

- Issue: Discrete models
  - Speech is continuously changing
  - How do we make observations? States?
- Solution: Discretize
  - “Time slices”: Make time discrete
  - Observations, States associated with time: Ot, Qt

Modelling Processes over Time

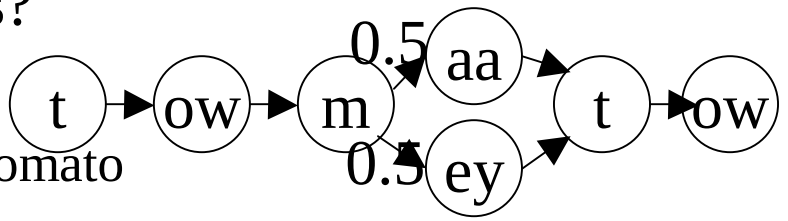
- Issue: New state depends on preceding states
  - Analyzing sequences
- Problem 1: Possibly unbounded # prob tables
  - Observation+State+Time
- Solution 1: Assume stationary process
  - Rules governing process same at all time
- Problem 2: Possibly unbounded # parents
  - Markov assumption: Only consider finite history
  - Common: 1 or 2 Markov: depend on last couple

Language Model

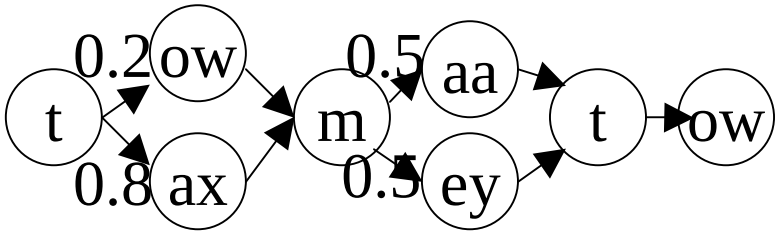
- Idea: some utterances more probable
- Standard solution: “n-gram” model
  - Typically tri-gram: P(wi|wi-1,wi-2)
    - Collect training data
      - Smooth with bi- & uni-grams to handle sparseness
  - Product over words in utterance

Acoustic Model

- P(signal|words)
  - words -> phones + phones -> vector quantiz'n
- Words -> phones
  - Pronunciation dictionary lookup
    - Multiple pronunciations?
      - Probability distribution
        - » Dialect Variation: tomato



        - » +Coarticulation



  - Product along path

Acoustic Model

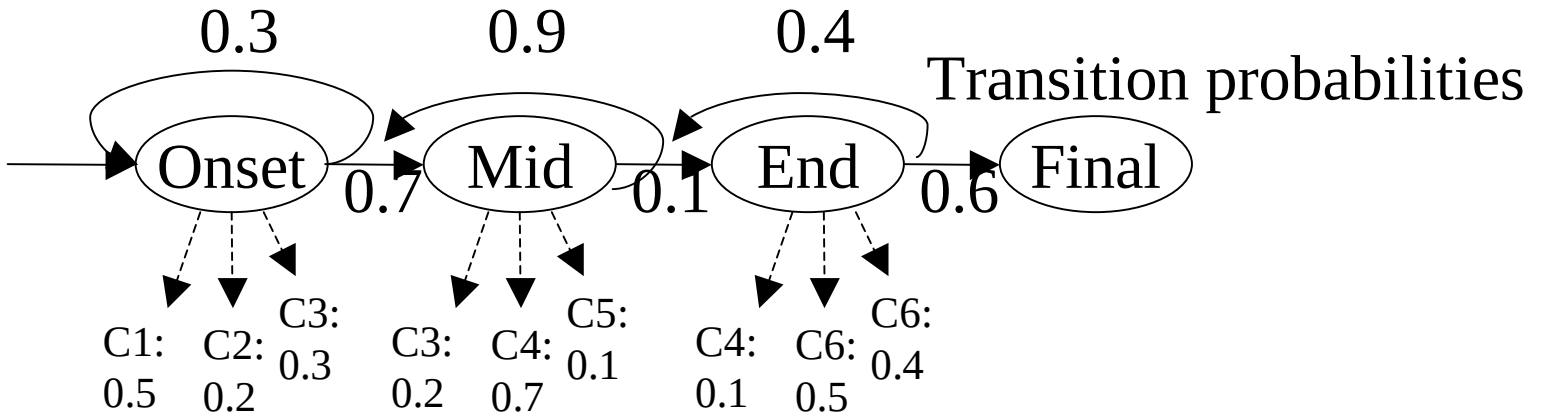
- P(signal| phones):
  - Problem: Phones can be pronounced differently
    - Speaker differences, speaking rate, microphone
    - Phones may not even appear, different contexts
  - Observation sequence is uncertain
- Solution: Hidden Markov Models
  - 1) Hidden => Observations uncertain
  - 2) Probability of word sequences =>
    - State transition probabilities
  - 3) 1st order Markov => use 1 prior state

Hidden Markov Models (HMMs)

- An HMM is:
  - 1) A set of states: $Q = q_o, q_1, ..., q_k$
  - 2) A set of transition probabilities: $A = a_{01}, ..., a_{mn}$
    - Where *aij* is the probability of transition *qi* -> *qj*
  - 3)Observation probabilities: $B = b_i(o_t)$
    - The probability of observing *ot* in state *i*
  - 4) An initial probability dist over states: π_i
    - The probability of starting in state *i*
  - 5) A set of accepting states

Acoustic Model

- 3-state phone model for [m]
  - Use Hidden Markov Model (HMM)



  - Probability of sequence: sum of prob of paths

Viterbi Algorithm

- Find BEST word sequence given signal
  - Best P(words|signal)
  - Take HMM & VQ sequence
    - => word seq (prob)
- Dynamic programming solution
  - Record most probable path ending at a state i
    - Then most probable path from i to end
    - O(bMn)

Viterbi Code

```
Function Viterbi(observations length T, state-graph) returns best-path
Num-states<-num-of-states(state-graph)
Create path prob matrix viterbi[num-states+2,T+2]
Viterbi[0,0]<- 1.0
For each time step t from 0 to T do
   for each state s from 0 to num-states do
      for each transition s' from s in state-graph
          new-score<-viterbi[s,t]*at[s,s']*bs'(ot)
          if ((viterbi[s',t+1]=0) || (viterbi[s',t+1]<new-score))
              then
                 viterbi[s',t+1] <- new-score
                 back-pointer[s',t+1]<-s
Backtrace from highest prob state in final column of viterbi[] & return
```

Enhanced Decoding

- Viterbi problems:
  - Best phone sequence not necessarily most probable word sequence
    - E.g. words with many pronunciations less probable
  - Dynamic programming invariant breaks on trigram
- Solution 1:
  - Multipass decoding:
    - Phone decoding -> n-best lattice -> rescoring (e.g. tri)

Enhanced Decoding: A*

- Search for highest probability path
  - Use forward algorithm to compute acoustic match
  - Perform **fast match** to find next likely words
    - Tree-structured lexicon matching phone sequence
  - Estimate path cost:
    - Current cost + underestimate of total
  - Store in priority queue
  - Search best first

Modeling Sound, Redux

- Discrete VQ codebook values
  - Simple, but inadequate
  - Acoustics highly variable
- Gaussian pdfs over continuous values
  - Assume normally distributed observations
    - Typically sum over multiple shared Gaussians
      - "Gaussian mixture models"
      - Trained with HMM model

$$b_j(o_t) = \frac{1}{\sqrt{(2\pi)\,|\sum j|}} e^{[(o_t-\mu_j)'\sum_j^{-1}(o_t-\mu_j)]}$$

Learning HMMs

- Issue: Where do the probabilities come from?
- Solution: Learn from data
  - Trains transition (aij) and emission (bj) probabilities
    - Typically assume structure
  - Baum-Welch aka forward-backward algorithm
    - Iteratively estimate counts of transitions/emitted
    - Get estimated probabilities by forward comput'n
      - Divide probability mass over contributing paths

Forward Probability

$$\alpha_t(i) = P(o_1, o_2, .., o_t, q_t = j \mid \lambda)$$

$$\alpha_j(1) = a_{1j} b_j(o_t), 1 < j < N$$

$$\alpha_j(t) = \left[\sum_{i=2}^{N-1} \alpha_j(t-1) a_{aj} \right] b_j(o_t)$$

$$P(O \mid \lambda) = \alpha_N(T) = \sum_{i=2}^{N-1} \alpha_i(T) a_{iN}$$

Backward Probability

$$\beta_i(t) = P(o_{t+1}, o_{t+2}, .., o_T \mid q_t = j, \lambda)$$

$$\beta_i(T) = a_{iN}$$

$$\beta_i(t) = \sum_{i=2}^{N-1} a_{ij} b_j(o_{t+1}) \beta_j(t+1)$$

$$P(O \mid \lambda) = \alpha_N(T) = \beta_i(T) = \sum_{j=2}^{N-1} a_{1j} b_j(o_1) \beta_j(1)$$

Re-estimating

- Estimate transitions from i->j

$$\tau_t(i,j) = \frac{\alpha_i(t) a_{ij} b_j(o_t) \beta_j(t+1)}{\alpha_N(T)}$$

$$\hat{a}_{ij} = \frac{\sum_{t=1}^{T-1} \tau_t(i,j)}{\sum_{t=1}^{T-1} \sum_{j=1}^{N} \tau_t(i,j)}$$

- Estimate observations in j

$$\sigma_j(t) = \frac{P(q_t = j, O \mid \lambda)}{P(O \mid \lambda)} = \frac{\alpha_j(t)\beta_j(t)}{P(O \mid \lambda)}$$

$$\hat{b}_j(v_k) = \frac{\sum_{t=1 s.t. o_t = v_k}^{T} \sigma_j(t)}{\sum_{t=1}^{T} \sigma_j(t)}$$

Does it work?

- Yes:
  - 99% on isolate single digits
  - 95% on restricted short utterances (air travel)
  - 80+% professional news broadcast
- No:
  - 55% Conversational English
  - 35% Conversational Mandarin
  - ?? Noisy cocktail parties

Speech Recognition as Modern AI

- Draws on wide range of AI techniques
  - Knowledge representation & manipulation
    - Optimal search: Viterbi decoding
  - Machine Learning
    - Baum-Welch for HMMs
    - Nearest neighbor & k-means clustering for signal id
  - Probabilistic reasoning/Bayes rule
    - Manage uncertainty in signal, phone, word mapping
- Enables real world application