CMSC 10500-1: Homework 3

(due on Friday July 2nd)

The magical rabbit colony

Long ago, there was a colony of magical rabbits. These rabbits would reproduce at an abominable rate. Each rabbit would take a month to mature. After the initial month, the rabbit produces one offspring every month. Also these magical rabbits never die. The colony started with one lone rabbit.

Let F_n denote the number of rabbits in the colony after n months.

1. **(2 pts)** Calculate F_1, F_2, F_3, F_4 and F_5.

2. **(3 pts)** Prove that for $n > 2$, $F_n = F_{n-1} + F_{n-2}$.

3. **(5 pts)** Write a simple scheme function `fib` which takes an input a number n and returns the value of F_n. Evaluate `(fib 30)` and notice how long it takes to calculate `(fib 30)`. If you are really adventurous you may try calculating `(fib 50)`.

4. **(5 pts)** Observe that the number of recursive calls needed to calculate `(fib 30)` is `(fib 30)`. Write another scheme function `fib2` to calculate F_n so that the number of recursive calls to calculate `(fib2 30)` is only 30, and evaluate `(log (fib2 1476))`.

   Hint: calculate the pair (F_{n+1}, F_n) *from* (F_n, F_{n-1}).

   Comments: `fib2` takes about n recursive calls to calculate F_n. One can actually write a `fib3` which requires less than $4 \log_2 n$ recursive calls to calculate F_n, using another clever trick.

Statistics

Let $a_1, \ldots, a_n$ be a sequence of numbers. The mean (denoted μ) and the Standard Deviation (denoted σ) are defined through the following formulae:

$$\mu := \frac{(a_1 + a_2 + \ldots + a_n)}{n}$$

$$\sigma := \sqrt{\frac{(a_1 - \mu)^2 + (a_2 - \mu)^2 + \ldots + (a_n - \mu)^2}{n}}$$

5. **(10 pts)** Write scheme function `mean` and `stddev` which consume lists of numbers and produce the mean and standard deviation of the input list respectively.