

Consider the following representation of terms in SML:

```
datatype term = T of (string * term list)
```

where the `string` is the operator name. It is possible to define strategy combinators for this term representation, where a strategy has the type

```
type strategy = term -> term option
```

and `NONE` denotes failure. For example,

```
fun <+ (s1, s2) t = (case s1 t  
  of NONE => s2 t  
    | someT => someT  
  (* end case *))
```

implements deterministic choice and

```
fun all s (T(f, args)) = let  
  fun try ([], l) = SOME(T(f, List.rev l))  
    | try (t::ts, l) = (case s t  
      of NONE => NONE  
        | SOME t' => try(ts, t'::l)  
      (* end case *))  
  in  
    try (args, [])  
  end
```

implements the `all` combinator.

1. Give the SML code for the `test` combinator.
2. Give the SML code for a generic congruence operator with the following specification:

```
val congruence : (string * strategy list) -> strategy
```

3. Give the SML code for the `one` combinator.
4. How would you implement the following rewrite rule in this framework:

$$\text{Plus}(\text{Times}(a, c), \text{Times}(b, c)) \implies \text{Times}(a, \text{Plus}(b, c))$$