Algorithms – CS-27200/37000 Homework – February 23, 2005 Instructor: László Babai Ry-164 e-mail: laci@cs.uchicago.edu

TA SCHEDULE: TA sessions are held in Ryerson-255, Tuesday and Thursday 5–6pm, Saturday 11am–noon, and (this is new) Wednesday after class 12:30–1:20 or 1:30–2:20 depending on demand. Indicate your interest in the Wednesday session to the instructor immediately after class. (The Wednesday evening sessions are discontinued.)

ADVICE. Take advantage of the TA sessions.

Check the class <u>website</u>, http://www.classes.cs.uchicago.edu/current/27200-1.

<u>READING.</u> Graduate students: study the proof of the Cook-Levin Theorem (NP-completeness of satisfiability) and applications to other NP-completeness proofs of CLIQUE, HAMILTON CYCLE, SUBSET SUM, 3-COLORABILITY (Ex. 34.3, page 1019).

DATES TO REMEMBER: Mon Mar 7: Midterm 2, Fri Mar 11: Last class. ATTENDANCE REQUIRED. Fri Mar 18, 10:30–12:30: Final Exam

16.1 (8 points) A problem A is NP-hard if all problems in NP have a Cookreduction to A. To demonstrate that a problem A is NP-hard, it suffices to reduce one known NP-complete problem to A.

The Longest path problem is defined as the set of pairs LONG = $\{(G, k) : G \text{ is a graph and } G \text{ has a path of length} \geq k\}$. A graph is Hamiltonian if it has a Hamilton cycle (a cycle passing through all vertices.) HAM is the set of Hamiltonian graphs. Use the fact that HAM \in NPC to prove (a) (6 points) that LONG is NP-hard; (b) (G only, 6 points) LONG \in NPC. If you solve (b), you also get a partial credit (3 points) toward (a). To get the full credit for (a), you need to give a very simple solution to (a).

- 16.2 (a) (6 points) Give a Karp-reduction from 3-COL to HALTING. (b) (G only, 5 points) Prove that no Karp-reduction from HALTING to 3-COL exists.
- 16.3 (a) (8 points, G only, due Monday Feb 28) Give a Cook-reduction from the problem of graph-3-colorability (a decision problem) to the problem of actually finding a 3-coloring. (b) (U, G, 3 points, due Monday Feb 28) Prove that no such Karp-reduction exists.