Algorithms – CS-27200/37000 Homework – February 28, 2005

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TA SCHEDULE: TA sessions are held in Ryerson-255, Tuesday and Thursday 5–6pm, Saturday 11am–noon, and (this is new) **Wednesday after class** 12:30–1:20 or 1:30–2:20 depending on demand. Indicate your interest in the Wednesday session to the instructor immediately after class. (The Wednesday evening sessions are discontinued.)

ADVICE. Take advantage of the TA sessions.

Check the class website, http://www.classes.cs.uchicago.edu/current/27200-1.

READING.

(G only): min-cost path in weighted DAG in linear time (text, Chap. 24.2, pp. 592–594) (due next class)

(U,G) B-trees (text, Chap. 18; especially 2-3-4-trees (text, p. 439 and 453)) (due for the final exam).

(G only): Finding the convex hull (text, Chap. 33.3, pp 974-957) (due for the final exam).

Graduate students: review the proof of the Cook-Levin Theorem (NP-completeness of satisfiability) and apllications to other NP-completeness proofs of CLIQUE, HAMILTON CYCLE, SUBSET SUM, 3-COLORABILITY (Ex. 34.3, page 1019).

DATES TO REMEMBER: Mon Mar 7: Midterm 2, Fri Mar 11: Last class. ATTENDANCE REQUIRED. Fri Mar 18, 10:30–12:30: Final Exam

18.1 (12 points) Give a Karp-reduction from 3-COL (graph 3-colorability) to 3-SAT. Prove that your function is indeed a Karp-reduction.

18.2 (8 points) Give a Karp-reduction from 3-COL to 4-COL (graph 4-colorability). Prove that your function is indeed a Karp-reduction.

18.3 (G only, 10 points) Let $F = C_1 \wedge C_2 \wedge \ldots \wedge C_m$ be a 3-CNF formula. Prove that at least $7m/8$ of the m clauses can be simultaneously satisfied. *Hint.* What is the expected number of clauses satisfied by a random assignment of Boolean values to the variables?

18.4 (Due Friday, March 4; 7 points) Draw a weighted undirected graph with a source vertex s and a target vertex t such that during the course of executing Dijkstra's algorithm, the cost $c(t)$ is reduced at least 5 times (from its initial value $c(t) = \infty$). Your graph should have as few edges as possible.

18.5 (G only, due Friday, March 4; 8 points) Louise designed a network of comparators (compare-exchange modules, like in sorting networks) which she claims will merge two sorted lists of n reals ($2n$ total). She implemented her design as a black box with $2n$ input wires and $2n$

output wires (appropriately labeled). She gives the box to Thelma for testing. The only thing Thelma can do is create an input, run it through the box, and check the output. Advise Thelma how to perform the testing using a polynomial number of test inputs. How many test inputs does Thelma need to use? Try to give as small an upper bound as possible.

RESEARCH PROBLEM. (No deadline, no points. The instructor does not know the answeer.) Prove [or disprove] that $O(n)$ tests do not suffice.