Introduction to Complexity Theory – CS-28100
Homework 3 – April 14, 2006
Instructor: Ketan Mulmuley   Ry-165B

HOMEWORK. Please **print your name on each sheet.** Please try to make your solutions readable.

This homework is due on **Wednesday, April 19** at the **beginning of the class.**

3.1 Given a function f, let $L_f = \{(x, f(x)) \mid x \in \Sigma^*, f \text{ defined on } x\}$. Then f is a total computable function iff L_f is recursive. Also f is a partial computable function iff L_f is recursively enumerable.

3.2 Prove that an infinite set is decidable iff it can be enumerated in increasing order by a one-to-one computable function. (Homework 3.4 from the book of Homer and Selman)

3.3 Let $L_1, L_2, \ldots, L_k$ be a collection of languages over Σ such that:

1. For all $i \neq j, L_i \cap L_j = \emptyset$; *i.e.* no string is in two languages.
2. $L_1 \cup L_2 \cup \cdots \cup L_k = \Sigma^*$; *i.e.* every string is in one of the languages.
3. Each of the languages L_i, for $i = 1, 2, \ldots, k$ is r.e.

Prove that each of the languages is therefore recursive. (Exercise 9.2.4 from the book of Hopcroft, Motwani & Ullman)

3.4 Tell (with proof) whether the recursive languages and/or the r.e. languages are closed under the following operations. You may give informal, but clear, constructions to show closure.

a. Union
b. Intersection
c. Concatenation
d. Kleene closure (star).
e. Homomorphism
f. Inverse homomorphism

(Exercise 9.2.6 from the book of Hopcroft, Motwani & Ullman)

3.5 Prove that every infinite c.e. (r.e.) set contains an infinite decidable (recursive, computable) subset. (Homework 3.6 from the book of Homer and Selman)