Introduction to Complexity Theory – CS-28100
Homework 4 – April 21, 2006
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HOMEWORK. Please **print your name on each sheet.** Please try to make your solutions readable.

This homework is due on **Friday, April 28** at the **beginning of the class.**

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4.1 Suppose we have a two-tape Turing machine, where one tape is an input tape, which cannot be written to, and the other tape is of a fixed finite length. Also, suppose that we can move right but cannot move left on the input tape.

Prove that any language recognizable by such a Turing machine is a regular language.

4.2 Prove that the statement of the previous exercise is false if we allow the head on the input tape to move in either direction. That is, define a language that can be recognized by such a machine, and prove that it is not regular. (Hint: use the Pumping Lemma.)

4.3 Prove that if a language $L$ can be recognized by a nondeterministic finite automaton, it can be recognized by a deterministic finite automaton.

4.4 Prove that the language $\{ww \mid w \in \{0,1\}^*\}$ cannot be generated by a context-free grammar. (Hint: use the Pumping Lemma for context-free grammars.)

4.5 Show that PCP is undecidable even if we limit the alphabet to $\Sigma = \{0,1\}$ by reducing PCP to this special case of PCP. (Exercise 9.4.2 from Hopcroft, Mulwani, Ullman)

4.6 Suppose we limited PCP to a one-symbol alphabet. Would this restricted case of PCP still be undecidable? (Exercise 9.4.3 from Hopcroft, Mulwani, Ullman)