Introduction to Complexity Theory – CS-28100
Homework 5 – May 10, 2006
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HOMEWORK. Please **print your name on each sheet.** Please try to make your solutions readable.

This homework is due on **Wednesday, May 17** at the **beginning of the class.**

5.1 Show that there is no algorithm that given as input a Turing machine M, where M defines a partial function of one variable, outputs a Turing machine M' such that M' defines a different partial function of one variable. (Hint: Suppose contrary, i.e. there exists a total function f and prove that it contradicts recursion theorem)
(Homework 3.11 from the book of Homer and Selman)

5.2 Let us define the set $K^{(w)}$ as follows:

$$K^{(w)} = \{\langle i, j\rangle \mid i \in K^{(j)}\}.$$

1. Show for every $j \geq 1$ that $K^{(j)}$ is many-one reducible to $K^{(w)}$.
2. Show that $K^{(w)}$ is not Turing reducible to $K^{(j)}$ for any $j \geq 1$.

(Homework 3.32 from the book of Homer and Selman)

5.3 Show that the functions $\lceil \log(n) \rceil, n^k, 2^n$ and $n!$ are space-constructible and, with the exception of $\lceil \log(n) \rceil$ time-constructible.
(Homework 5.2 from the book of Homer and Selman)

5.4 If one of the following classes is included in another state which and explain why. If one of the inclusions is a proper inclusion, then state that and explain why: DTIME(n^2), DSPACE(n^8), NTIME(n^2), NSPACE(n^5).
(Homework 5.12 from the book of Homer and Selman)

5.5 Prove that NP is not included in DTIME(n^k) for any fixed $k \geq 1$.
(Homework 5.16 from the book of Homer and Selman)