Introduction to Complexity Theory – CS-28100
Homework 6 – May 21, 2006
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HOMEWORK. Please **print your name on each sheet.** Please try to make your solutions readable.

This homework is due on **Friday, May 26** at the **beginning of the class.**

6.1 For $L \subseteq \Sigma^*$, let $\text{Tally}(L) = \{1^{n(w)} | w \in L\}$, where $n(w)$ indicates the number associated with the word w. Prove that $\text{Tally}(L) \in \text{P}$ if and only if $L \in \text{E}$.
(Part of Homework 5.9 from the book of Homer and Selman)

6.2 Let $\text{CLIQUE} = \{(G, k) | G \,\text{has a clique of size}\, k\}$. We know CLIQUE is in NP, but is NP-complete. Show that some infinite subset of CLIQUE belongs to P.
(Homework 6.16 from the book of Homer and Selman)

6.3 Define $\text{MAXCLIQUE} \subset \text{CLIQUE}$ as follows:
$\text{MAXCLIQUE} = \{(G, m) \in \text{CLIQUE} | (\forall k > m)(G, k) \notin \text{CLIQUE}\}$.
Show that MAXCLIQUE is in NP.

6.4 Show that $\text{PRIME} = \{p | p \,\text{is prime}\}$ is in NP. Indeed it is shown that PRIME is in P.

6.5 Using the previous exercise show that FCTR is in NP.
$\text{FCTR} = \{(n; k, p_1, \alpha_1, \ldots, p_k, \alpha_k) | n = p_1^{\alpha_1} \ldots p_k^{\alpha_k}, (\forall i) p_i \in \text{PRIME}\}$