Algorithms CMSC-37000 First Quiz. January 16, 2007
Instructor: László Babai

Name: ______________________________

Show all your work. **Do not use book, notes, or scrap paper.** Write your answers in the space provided. When describing an algorithm in pseudocode, **explain the meaning of your variables** (in English). This quiz contributes 6% to your course grade.

1. (12 points) Let $a_n = \ln(n!)$ and $b_n = n((\ln n) - 1)$. Let $d_n = a_n - b_n$. Asymptotically evaluate d_n. Find constants r, s, t such that $d_n \sim rn^s(\ln n)^t$. Determine the values r, s, t. Prove your answer.

2. (9 points) A complete binary tree has n nodes. Determine the **exact** height of the tree. **Prove** your answer.

3. (15 points) Describe in pseudocode the INCREASE-KEY(x, *newkey*) operation in a heap. (x is the name of a node; *newkey* is the value with which we replace key(x). We assume *newkey* $>$ key(x).)

4. (24 points) n coins of various values are arranged on a line. Two players take turns to remove a coin from an end of the line until all coins have been removed. Each player in their turn removes exactly one coin and the coin must come either from the right end or the left end of the line so the remaining coins remain contiguous. Each player wants to maximize the total value of the coins collected. Find the optimal move for the first player in $O(n^2)$ steps. (Each player plays optimally.) Hint: dynamic programming.

   Example: if the coin values are 10, 25, 10, 1 cents then Player 1 should pick the penny because picking the dime would permit Player 2 to pick the quarter. Now out of 10, 25, 10, no matter what Player 2 picks, the quarter will go to Player 1. So if each player plays optimally, Player 1 will collect 26 cents, while Player 2 gets 20.