Homework 2

due Friday Oct 19 in class

1. Convert the following DFA to a regular expression, using the state-elimination technique.

	0	1
$\rightarrow *p$	s	p
q	p	s
r	r	q
s	q	r

2. Convert the following regular expressions to NFA's with ϵ-transitions.

   (a) $\mathbf{01}^*$

   (b) $(\mathbf{0}+\mathbf{1})\mathbf{01}$

   (c) $\mathbf{00}(\mathbf{0}+\mathbf{1})^*$

3. Prove that the following are not regular languages.

   (a) $\{0^n1^n \mid n \geq 1\}$

   (b) The set of strings of balanced parantheses. These are the strings of characters "(" and ")" that can appear in a well-formed arithmetic expression.

   (c) $\{0^n10^n \mid n \geq 1\}$

   (d) $\{0^n1^m2^n \mid n, m \in \mathbb{N}\}$

   (e) $\{0^n1^m \mid n \leq m\}$

   (f) $\{0^n1^{2n} \mid n \geq 1\}$

4. Prove that the following are not regular languages.

   (a) $\{0^n \mid n$ is a perfect square$\}$

   (b) $\{0^n \mid n$ is a perfect cube$\}$

   (c) $\{0^n \mid n$ is a power of 2$\}$

   (d) The set of strings of 0's and 1's whose length is a perfect square.

   (e) The set of strings of 0's and 1's that are of the form ww, that is, some string repeated.

   (f) The set of strings of 0's and 1's that are of the form ww^R, that is, some string followed by its reverse.

   (g) The set of strings of 0's and 1's of the form $w\overline{w}$, where $\overline{w}$ is formed from w by replacing 0's by 1's , and vice-versa.

   (h) The set of strings of the form $w1^n$, where w is a string of 0's and 1's of length n.

5. Show that the regular languages are closed under the following operations:

   (a) $min(L) = \{w \mid w \text{ is in } L, \text{ but no proper prefix of } w \text{ is in } L\}$

   (b) $max(L) = \{w \mid w \text{ is in } L \text{ and for no } x \text{ other than } \epsilon \text{ is } wx \text{ in } L\}$

   (c) $init(L) = \{w \mid \text{for some } x, wx \text{ is in } L\}$

   Hint: start with a DFA for L and perform a construction to get the desired language.