Homework 6

due Monday Nov 26 in class

1. Deisgn a Turing machine that takes as input a number N and adds 1 to it in binary. To be precise, the tape initially contains a \$ followed by N in binary. The tape head is initially scanning the \$ in state q_0. Your TM should halt with $N+1$, in binary, on its tape, scanning the leftmost symbol of $N+1$, in state q_f. You may destroy the \$ in creating $N+1$, if necessary. For instance, $q_0\$10011 \vdash^* \$q_f10100$, and $q_0\$11111 \vdash^* q_f100000$.

   (a) Give the transitions of your TM and explain the purpose of each state.

   (b) Show the sequence of ID's of your TM when given input \$111.

2. Consider the nondeterministic Turing machine

$$M = (\{q_0, q_1, q_2, q_f\}, \{0, 1\}, \{0, 1, B\}, \delta, q_0, B, \{q_f\})$$

   Informally but clearly describe the language $L(M)$ if δ consists of the following sets of rules: $\delta(q_0, 0) = \{(q_0, 1, R), (q_1, 1, R)\}; \delta(q_1, 1) = \{(q_2, 0, L)\}$; $\delta(g_2, 1) = \{(q_0, 1, R)\}; \delta(q_1, B) = \{(q_f, B, R)\}$.

3. A *two-dimensional* Turing machine has the usual finite-state control but a tape that is a two-dimensional grid of cells, infinite in all directions. The input is placed on one row of the grid, with the head at the left end of the input and the control in the start state, as usual. Acceptance is by entering a final state, also as usual. The head of a two-dimensional Turing machine moves one square up (U), down (D), left (L) or right (R). Prove that the languages accepted by two-dimensional Turing machines are the same as those accepted by ordinary TM's.

4. If L is a language, and a is a symbol, then L/a, the *quotient* of L and a, is the set of strings w such that wa is in L. For example, if $L = \{a, aab, baa\}$, then $L/a = \{\epsilon, ba\}$. Prove that the DPDA languages are closed under the operation L/a. *Hint:* Let P be a DPDA for L. You need to construct a DPDA P' for L/a. Modify P by replacing each of its stack symbols X by all possible pairs (X, S), where S is a set of states. If P has stack $X_1, \ldots, X_n$, then the constructed DPDA for L/a has stack $(X_1, S_1), \ldots, (X_n, S_n)$, where each S_i is the set of states q such that P, started in ID $(q, a, X_i \cdots X_n)$ will accept.