Homework 3 Solutions

DISCLAIMER: The solutions presented below are incomplete and might be insufficient to get full grade on the homework. They do not model acceptable solutions, but rather present an idea of how a certain problem can be approached. A diligent student should be able to work out complete solutions. Please report any mistakes that you find to the instructor and TA(s).

1. Let A be a DFA for L. Mark all states reachable from the start state (can use either DFS or BFS for this). $L(A) = L = \Sigma^*$ if and only if all the marked nodes are accepting.

2. Let $A_i = (Q_i, \Sigma_i, \delta_i, s_i, F_i)$ be a DFA for L_i, where $i \in \{1, 2\}$. Use product construction to obtain DFA A for $L_1 \cap L_2$, i. e.,

$$A = (Q_1 \times_Q 2, \Sigma_1 \cup \Sigma_2, \delta, (s_1, s_2), F_1 \times F_2),$$

where $\delta(a, b) = (\delta_1(a), \delta_2(b))$ for $a \in \Sigma_1$, $b \in \Sigma_2$. Then $L_1 \cap L_2 \neq \emptyset$ if and only if some final state is reachable from (s_1, s_2) in A.

3. (a)

B	X							
C	X	X						
D		X	X					
E	X		X	X				
F	X	X		X	X			
G		X	X		X	X		
H	X		X	X		X	X	
I	X	X		X	X		X	X
	A	B	C	D	E	F	G	H

(b)

	0	1
$\rightarrow \{A, D, G\}$	$\{B, H, E\}$	$\{B, H, E\}$
$\{B, H, E\}$	$\{C, F, I\}$	$\{C, F, I\}$
$*\{C, F, I\}$	$\{A, D, G\}$	$\{B, H, E\}$

4. (a)

$$S \;\rightarrow\; 01 | 0S1$$

(b)

$$
\begin{aligned}
S &\;\rightarrow\; EC | AF \\
A &\;\rightarrow\; aA | \epsilon \\
B &\;\rightarrow\; bB | \epsilon \\
C &\;\rightarrow\; cC | \epsilon \\
E &\;\rightarrow\; aEb | aA | bB \\
F &\;\rightarrow\; bFc | bB | cC
\end{aligned}
$$

(c)

$$
\begin{aligned}
S &\rightarrow A|B|AB|BA \\
A &\rightarrow CAC|a \\
B &\rightarrow CBC|b \\
C &\rightarrow a|b
\end{aligned}
$$

(d)

$$
S \rightarrow \epsilon|S0S0S1S|S0S1S0S|S1S0S0S
$$

5.

a) 00101

b) 1001

c) 00011