DISCLAIMER: The solutions presented below are incomplete and might be insufficient to get full grade on the homework. They do not model acceptable solutions, but rather present an idea of how a certain problem can be approached. A diligent student should be able to work out complete solutions. Please report any mistakes that you find to the instructor and TA(s).

1. Assume that L is a CFL. Let n be given by the pumping lemma. Choose $z \in L$ s.t. $|z| \geq n$. Then by the pumping lemma $z = uvwxy$ s.t. $|vwx| \leq n$ and $\forall k \geq 0, uv^k wx^k y \in L$. Then find a contradiction.

   (a) Let $z = a^n b^{n+1} c^{n+2}$. If there are no c's in vwx then uv^3wx^3y will have at least $n+2$ a's or b's and only $n+2$ c's, so it cannot be in L. If vwx contains at least one c then it cannot contain an a since $|vwx| \leq n$. If we pump down to v^0wx^0, then uwy has at most $2n+1$ b's and c's and n a's, so there can't be both more than n b's and more than n c's.

   (b) Let $z = a^n b^n c^n$. If v and x do not contain any c's then pumping down will give us the contradiction because n will be $< i$. If vwx is all c's, pumping up will give us the contradiction because $i > n$. If x contains a c and v at least one b, then there cannot be any a's in vwx so pumping either way will break the condition that $|a| = |b|$. The same contradiction occurs if vwx is all a's or all b's.

   (c) Let $z = 0^p$ for some prime number $p \geq n$. Let $|vx| = m \geq 1$. Then $uv^iwx^iy = 0^{p+m(i-1)}$. But if $i = p$ then this is $0^{p+pm} = 0^{p(m+1)}$, which is clearly not prime.

2. (a) L_1 grammar

$$S \to AB \tag{1}$$
$$A \to aAbb|\epsilon \tag{2}$$
$$B \to cB|\epsilon \tag{3}$$
$$\tag{4}$$

   L_2 grammar

$$S \to AB \tag{5}$$
$$A \to Aa|\epsilon \tag{6}$$
$$B \to bBcc|\epsilon \tag{7}$$
$$\tag{8}$$

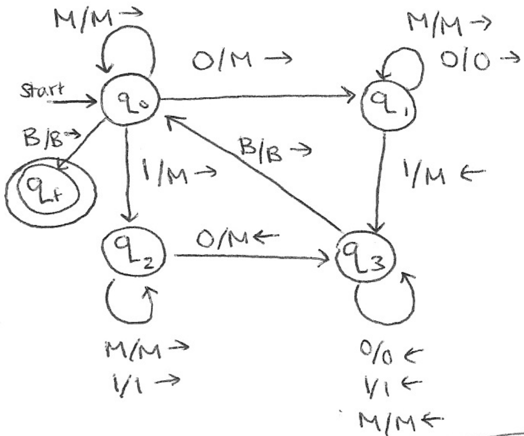
   (b) $L_1 \cap L_2$ is not a CFL. $L_1 \cap L_2 = \{a^n b^{2n} c^{4n} | n \geq 0\}$. Apply the pumping lemma to string $z = a^n b^{2n} c^{4n}$.

3. (a) Let L be a CFL and n given by the pumping lemma. Then if there is a string of length greater than n the laguage is infiinite. Test all strings of length n to $2n-1$ and if there exists such a string in the language, the lanugage is infinite, else it is finite.

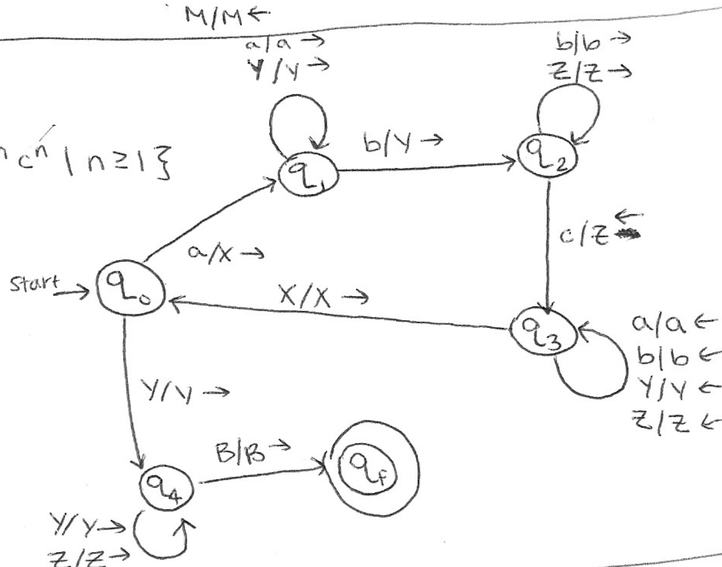
(b) If L is inifinite there certainly exists 100 strings in the language. Otherwise, count the number of strings in the language of length 0 to n, n given by the pumping lemma, and if there are at least 100 such strings answer yes otherwise no.

(c) Put the CFG in Chomsky Normal Form, which allows us to represent the language as a tree. Use a breadth first search to search the left branches of the tree and ruture true if we see an A, return false if we exhaust the search space or end up in a cycle.

4a) L = set of strings where number of 0's = number of 1's



4b) $L = \{ a^n b^n c^n \mid n \geq 1 \}$



4c) $L = \{ ww^R \mid w \in \{0,1\}^* \}$