Homework 2

Exercise 0.1 *Prove that the following are not regular languages.*

1. $\{0^n : n \text{ is a perfect square}\}$.
2. $\{0^n : n \text{ is a perfect cube}\}$.
3. $\{0^n : n \text{ is a power of } 2\}$.
4. The set of strings of 0's and 1's whose length is a perfect square.
5. The set of strings of 0's and 1's that are of the form ww, that is, some string repeated.
6. The set of strings of 0's and 1's that are of the form ww^R, that is, some string followed by its reverse. (The reversal of a string $a_1a_2 \dots a_n$ is the string written backwards, that is, $a_n a_{n-1} \dots a_1$.
7. The set of strings of 0's and 1's of the form $w\overline{w}$, where $\overline{w}$ is formed from w by replacing all 0's by 1's, and vice-versa; e.g. $\overline{011} = 100$, and 011100 is an example of a string in the language.
8. The set of strings of the form $w1^n$, where w is a string of 0's and 1's of length n.

Exercise 0.2 *Show that the regular languages are closed under the following operations: (Hint: it is easiest to start with a DFA for L and perform a construction to get the desired language.)*

1. $min(L) = \{w : w \text{ is in } L, \text{ but no proper prefix of } w \text{ is in } L\}$.
2. $max(L) = \{w : w \text{ is in } L \text{ and for no } x \text{ other than } \varepsilon \text{ is } wx \text{ in } L\}$.
3. $init(L) = \{w : \text{for some } x, wx \text{ is in } L\}$.

Exercise 0.3 *Give an algorithm to tell whether two regular languages L_1 and L_2 have at least one string in common.*

* Exercises above are from *Introduction to Automata Theory, Languages, and Computation, 3rd Edition*: Exercises 4.1.2, 4.2.6, 4.3.4