

### Homework 3

**Exercise 0.1** Suppose  $L$  is a regular language with alphabet  $\Sigma$ . Give an algorithm to tell whether  $L$  contains at least 100 strings.

**Exercise 0.2** If  $w = a_1a_2 \dots a_n$  and  $x = b_1b_2 \dots b_n$  are strings of the same length, define  $\text{alt}(w, x)$  to be the string in which the symbols of  $w$  and  $x$  alternate, starting with  $w$ , that is,  $a_1b_1a_2b_2 \dots a_nb_n$ . If  $L$  and  $M$  are languages, define  $\text{alt}(L, M)$  to be the set of strings of the form  $\text{alt}(w, x)$ , where  $w$  is any string in  $L$  and  $x$  is any string in  $M$  of the same length. Prove that if  $L$  and  $M$  are regular, so is  $\text{alt}(L, M)$ .

**Exercise 0.3** If  $L$  is a language, and  $a$  is a symbol, then  $a \setminus L$  is the set of strings  $w$  such that  $aw$  is in  $L$ . For example, if  $L = \{a, aab, baa\}$ , then  $a \setminus L = \{\varepsilon, ab\}$ . Prove that if  $L$  is regular, so is  $a \setminus L$ .

**Exercise 0.4** Given the DFA of Table 1, draw the table of distinguishabilities for this automaton and construct the minimum-state equivalent DFA.

Table 1: DFA to minimize

	0	1
$\rightarrow A$	B	E
B	C	F
*C	D	H
D	E	H
E	F	I
*F	G	B
G	H	B
H	I	C
*I	A	E

\* Exercises above are from *Introduction to Automata Theory, Languages, and Computation, 3rd Edition*: Exercises 4.2.3, 4.2.7, 4.3.2, 4.4.2