Homework 7

Exercise 0.1 *Design a Turing machine that takes as input a number N and adds 1 to it in binary. To be precise, the tape initially contains a \$ followed by N in binary. The tape head is initially scanning the \$ in state q_0. Your TM should halt with $N+1$, in binary, on its tape, scanning the leftmost symbol of $N+1$, in state q_f. You may destroy the \$ in creating $N+1$, if necessary. For instance, $q_0\$10011 \vdash^* \$q_f10100$, and $q_0\$11111 \vdash^* q_f100000$.*

1. *Give the transitions of your Turing machine, and explain the purpose of each state.*
2. *Show the sequence of ID's of your TM when given input \$111.*

Exercise 0.2 *Informally but clearly describe nondeterministic Turing machines - multitape if you like - that accept the following languages. Try to take advantage of nondeterminism to avoid iteration and save time in the nondeterministic sense. That is, prefer to have your NTM branch a lot, while each branch is short.*

1. *The language of all strings of 0's and 1's that have some string of length 100 that repeats, not necessarily consecutively. Formally, this language is the set of strings of 0's and 1's of the form $wxyxz$, where $|x| = 100$, and w, y, and z are of arbitrary length.*
2. *The language of all strings of the form $w_1\#w_2\#\ldots\#w_n$, for any n, such that each w_i is a string of 0's and 1's, and for some j, w_j is the integer j in binary.*
3. *The language of all strings of the same form as (b), but for at least two values of j, we have w_j equal to j in binary.*

Exercise 0.3 *Consider the nondeterministic Turing machine*

$$M = (\{q_0, q_1, q_2, q_f\}, \{0,1\}, \{0,1,B\}, \delta, q_0, B, \{q_f\})$$

Informally but clearly describe the language $L(M)$ if δ consists of the following sets of rules: $\delta(q_0, 0) = \{(q_0, 1, R), (q_1, 1, R)\}; \delta(q_1, 1) = \{(q_2, 0, L)\}; \delta(q_2, 1) = \{(q_0, 1, R)\}; \delta(q_1, B) = \{(q_f, B, R)\}$.

Exercise 0.4 *A k-head Turing machine has k heads reading cells of one tape. A move of this TM depends on the state and on the symbol scanned by each head. In one move, the TM can change state, write a new symbol on the cell scanned by each head, and can move each head left, right, or keep it stationary. Since several heads may be scanning the same cell, we assume the heads are numbered 1 through k, and the symbol written by the highest numbered head scanning a given cell is the one that actually gets written there. Prove that the languages accepted by k-head Turing machines are the same as those accepted by ordinary TM's.*

* Exercises above are from *Introduction to Automata Theory, Languages, and Computation, 3rd Edition*: Exercises 8.2.3, 8.4.3, 8.4.4, 8.4.9