

1. Prove that $\sqrt{\mathbf{u} \cdot \mathbf{u}} = \|\mathbf{u}\|$ for any vector \mathbf{u} .
2. Let $\mathcal{B} = \{\mathbf{b}_1, \dots, \mathbf{b}_n\}$ be an orthonormal basis and let $\mathbf{B} = [\mathbf{b}_i]$ be the matrix whose columns are the basis vectors.
Prove that $\mathbf{B}^{-1} = \mathbf{B}^T$.

3. Prove that for any two non-zero vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$,

$$(\mathbf{u} \cdot \mathbf{v})^2 + \|\mathbf{u} \times \mathbf{v}\|^2 = \mathbf{u}^2 \mathbf{v}^2$$

(where the notation \mathbf{u}^2 means $\mathbf{u} \cdot \mathbf{u}$).

4. Prove that for any three vectors $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$,

$$(\mathbf{u} \times \mathbf{v}) \times \mathbf{w} = (\mathbf{u} \cdot \mathbf{w})\mathbf{v} - (\mathbf{v} \cdot \mathbf{w})\mathbf{u}$$

5. Consider two unit vectors, \mathbf{u} and \mathbf{v} . The *linear interpolation* between these vectors is defined to be

$$\text{lerp}(\mathbf{u}, t, \mathbf{v}) = (1 - t)\mathbf{u} + t\mathbf{v}$$

where $0 \leq t \leq 1$. While this operation works well when the vectors represent positions, it does not work well when the vectors represent directions, since the angle between \mathbf{u} and $\text{lerp}(\mathbf{u}, t, \mathbf{v})$ is not a linear function of t .

Give pseudocode for a function *slerp*($\mathbf{u}, t, \mathbf{v}$), where $0 \leq t \leq 1$, that returns a unit vector \mathbf{w} , such that the angle between \mathbf{u} and \mathbf{w} is a linear function of t .