| CMSC 23700 | Introduction to Computer Graphics | Homework 3 |
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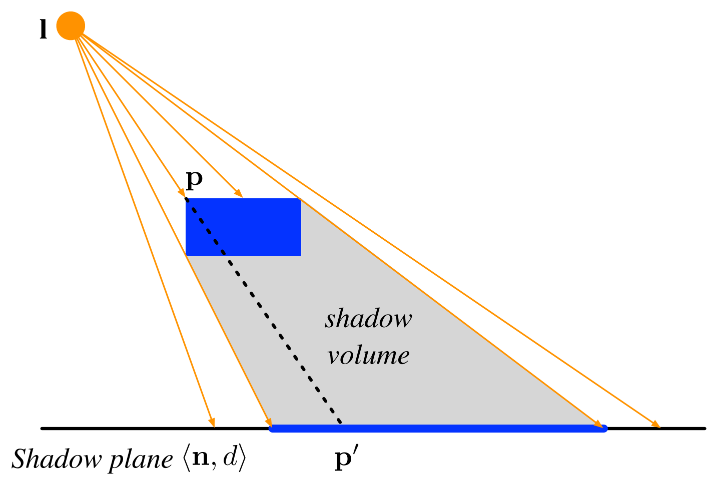
| Autumn 2015 | | Due November 10 |

1. **Note:** This question is based on material in the reading that we did not cover in class.

   The term Z-fighting is used to describe the situation when two primitives are mapped to the same Z-buffer value. Suppose you have an application with a near plan of 10 meters, a far plane of 100 kilometers ($10^5$ meters), and a minimum feature size of 1 meter. How many bits of Z-buffer do you need to avoid Z-fighting? What if the near plane is at 1 meter?

2. Planar projective shadowing is a simple technique for rendering shadows cast onto a planar surface. Consider the following situation

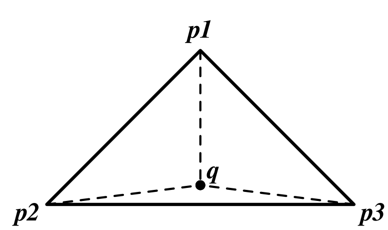



   where $\mathbf{l}$ is the position of a point light, $P = \langle \mathbf{n}, d \rangle$ is the shadow plane, and $\mathbf{p}$ is a vertex of the shadow caster.

   (a) Derive the point $\mathbf{p}'$ that is the projection of $\mathbf{p}$ onto the plane $P$.

   (b) The result from part (a) can be represented as a $4 \times 4$ transformation matrix $\mathbf{M}$ that maps points on the shadow caster to *homogeneous* coordinates on the plane. What is $\mathbf{M}$?

3. One way to make level-of-detail (LOD) transitions is to use an $\alpha$ *fade*, where you lerp the $\alpha$ channel to blend the two LODs. For example, assume that you have a triangle $\triangle\langle \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3 \rangle$ and a vertex $\mathbf{q}$ that splits the triangle into two triangles $\triangle\langle \mathbf{p}_1, \mathbf{p}_2, \mathbf{q} \rangle$ and $\triangle\langle \mathbf{p}_1, \mathbf{q}, \mathbf{p}_3 \rangle$ as follows:



At the coarse LOD, we just render $\triangle\langle \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\rangle$, and at the fine LOD, we render both $\triangle\langle \mathbf{p}_1, \mathbf{p}_2, \mathbf{q}\rangle$ and $\triangle\langle \mathbf{p}_1, \mathbf{q}, \mathbf{p}_3\rangle$, but in between we render all three triangles and use alpha blending to smooth the transition.

Assuming that $0 \leq t \leq 1$, give the blending equation that describes how to combine the two images as a function of $t$. It should be the case that when $t = 0$, just the coarse LOD is rendered and when $t = 1$, just the fine LOD is rendered.

4. An *axis-aligned bounding box* (AABB) in 2D is defined by four scalar values:

$$\langle minX, maxX, minY, maxY\rangle$$

We use $\langle 1, -1, 1, -1\rangle$ to denote the empty AABB. Let

$$bb_1 = \langle minX_1, maxX_1, minY_1, maxY_1\rangle$$

and

$$bb_2 = \langle minX_2, maxX_2, minY_2, maxY_2\rangle$$

be two *non-empty* AABBs.

(a) What is the minimum AABB that contains the union of $bb_1$ and $bb_2$?

(b) What is the minimum AABB that contains the intersection of $bb_1$ and $bb_2$?

(c) What is the minimum AABB that contains the difference of $bb_1$ and $bb_2$ (*i.e.*, $bb_1 \setminus bb_2$).?