Mathematics

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Program of Study

The Department of Mathematics provides an environment of research and comprehensive instruction in mathematics and applied mathematics at both undergraduate and graduate levels. Both a B.A. and a B.S. program in mathematics are offered, including a B.S. in applied mathematics. Students may also earn a B.S. degree with a Specialization in Computer Science or with a Specialization in Economics.

The requirements for a degree in mathematics or in applied mathematics express the educational intent of the Department of Mathematics; they are drawn with an eye toward the cumulative character of an education based in mathematics, the present emerging state of mathematics, and the scholarly and professional prerequisites of an academic career in mathematics.

Requirements for each bachelor's degree look to the advancement of students' general education in modern mathematics and their knowledge of its relation with the other sciences (B.S.) or with the other arts (B.A.).

Descriptions of the detailed requirements that give meaning to these educational intentions follow. Students should understand that any particular degree requirement can be modified if persuasive reasons are presented to the department; petitions to modify requirements are submitted in person to the director of undergraduate studies or the departmental counselor.

Placement. At what level does an entering student begin mathematics at the University of Chicago? This question is answered individually for students on the basis of their performance on one of two placement tests in mathematics administered during Orientation in September: either a precalculus mathematics placement test or a calculus placement test. Scores on the *mathematics* placement test determine the appropriate beginning mathematics course for each student: a precalculus course (MATH 10500) or one of three other courses (MATH 11200, MATH 13100, or MATH 15100). Students who wish to begin at a level higher than MATH 15100 *must* take the *calculus* placement test, unless they receive Advanced Placement credit as described in the following paragraphs.

Students who submit a score of 5 on the AB Advanced Placement exam in mathematics or a score of 4 on the BC Advanced Placement exam in mathematics receive credit for MATH 15100. Students who submit a score of 5 on the BC Advanced Placement exam in mathematics receive credit for MATH 15100 and 15200. Currently no course credit is offered in the Mathematics Department at Chicago for work done in an International Baccalaureate Program or for British A-level or for O-level examinations.

Students with suitable achievement on the calculus placement test are invited to begin with Honors Calculus (MATH 16100) or beyond. Excellent scores on the calculus placement test may give placement credit for one, two, or three quarters of calculus. The strong recommendation from the department is that students who have AP credit for one or two quarters of calculus enroll in Honors Calculus (MATH 16100) when they enter as first year students. This course builds on the strong computational background provided in AP courses and best prepares entering students for further study in mathematics.

Admission to Honors Analysis (MATH 20700) is by invitation only to those first-year students who show an exceptional performance on the calculus placement test or to those sophomores who receive a strong recommendation from their instructor in MATH 16100-16200-16300.

Program Requirements

Undergraduate Programs. Five bachelor's degrees are available in the Department of Mathematics: the B.A. in mathematics, the B.S. in mathematics, the B.S. in applied mathematics, the B.S. in mathematics with specialization in computer science, and the B.S. in mathematics with specialization in economics. Programs qualifying students for the degree of B.A. provide more elective freedom, while programs qualifying students for the degrees of B.S. require more emphasis in the physical sciences. All degree programs, whether qualifying students for a degree in mathematics or in applied mathematics, require fulfillment of the College's general education requirements. The general education sequence in the physical sciences must be selected from either first-year basic chemistry or first-year basic physics. The courses that make up the concentration program include at least nine courses in mathematics (detailed descriptions follow for each degree), plus at least five courses within the Physical Sciences Collegiate Division (PSCD) but outside mathematics, one of which completes the three-quarter sequence in basic chemistry or basic physics, and at least two others which should form a sequence of courses from a single department. These latter courses must be chosen from astronomy, chemistry, computer science, physics (12000s or above), geophysical sciences, or statistics (22000 or above). We particularly call attention to two degree programs that are described in more detail in the following paragraphs: (1) the B.S. in mathematics with specialization in computer science, and (2) the B.S. in mathematics with specialization in economics.

NOTE: Mathematics concentrators may use AP credit for chemistry or physics to meet their general education physical sciences requirement or the physical sciences component of the concentration. However, no credit designated simply as "physical science," from AP examinations or from the College's physical sciences placement or accreditation examination, may be used in their general education requirement or in the concentration.

Mathematics concentrators are required to complete: a 10000-level sequence in calculus (or to demonstrate equivalent competence on the calculus placement test); a three-quarter sequence in analysis (MATH 20300-20400-20500 or 20700-20800-20900); and two quarters of an algebra sequence (MATH 25400-25500 or 25700-25800). MATH 25700-25800-25900 is designated as an honors section of Basic Algebra. Registration for this course is the option of the individual student. Consultation with the departmental counselor is strongly advised. The normal procedure is to take calculus in the first year, analysis in the second, and algebra in the third.

Concentrators in mathematics or applied mathematics may take any 20000-level mathematics courses elected beyond concentration requirements for a grade of *P*. However, a grade of *C*- or higher must be earned in *each* calculus, analysis, or algebra course, and an overall grade average of *C* or higher must be earned in the remaining mathematics courses that are used to meet concentration requirements. Courses in the PSCD that are used to meet concentration requirements in mathematics must be taken for quality grades.

Students taking a bachelor's degree in mathematics or in applied mathematics should note that by judicious employment of courses from another field for extradepartmental requirements or for electives, a minor field can be developed that is often in itself a sufficient base for graduate or professional work in their field. The field of statistics furnishes a notable example: the core programs are STAT 24200 and 25100 for probability theory and STAT 24400 and 24500 for statistical theory. For an emphasis on statistical methods, students would add STAT 22200, 22400, or 22600; while for an emphasis on probability they would add STAT 31200 or perhaps STAT 38100-38200.

What is noted here for statistics can also be applied to computer science (see following section), chemistry, geophysical sciences, physics, biophysics, or theoretical biology. While these remarks apply to all bachelor's degree programs in the Department of Mathematics, their force is particularly evident in programs looking to bachelor's degrees in applied mathematics, where minor fields are strongly urged.

Degree Programs in Mathematics. Candidates for the B.A. and B.S. in mathematics all take a sequence in basic algebra. Candidates for the B.S. degree must take the three-quarter algebra sequence (MATH 25400-25500-25600 or MATH 25700-25800-25900), whereas B.A. candidates may opt for a two-quarter sequence (MATH 25400-25500 or MATH 25700-25800).

The remaining mathematics courses needed in the concentration programs (three for the B.A., two for the B.S.) must be selected, with due regard for prerequisites, from the following list of mathematics courses. STAT 25100 also meets the requirement. B.A. candidates may include MATH 25600 or 25900.

17500	21100	24100	24200	24300	26100	26200	26300
27000	27200	27300	27400	27500	27700	27800	27900
28000	28100	28400	29200	29700*	30000	30100	30200
30300	30900	31000	31200	31300	31400	31700	31800
31900	32500	32600	32700				

* as approved

B.S. candidates are further required to select a minor field, which consists of an additional three-course sequence, which is outside the Department of Mathematics but within the Physical Sciences Collegiate Division, chosen in consultation with the departmental counselor.

Summary of Requirements: Mathematics

General Education		CHEM 11101-11201/11102-11202 or equivalent†, or PHYS12100-12200 or higher† MATH 13100-13200, 15100-15200, or 16100-16200†
Concentration	1	CHEM 11301/11302 or equivalent†, or PHYS 12300 or higher†
	1	MATH 13300, 15300, or 16300†
	3	MATH 20300-20400-20500 or 20700-20800-20900
	2	courses in mathematics chosen from an approved list
	4	courses within the PSCD but outside of mathematics, at least two of which should be taken in a single department*

plus the following requirements:

	B.A.		**B.S.**
2	MATH 25400-25500 or 25700-25800	3	MATH 25400-25500-25600 or 25700-25800-25900
1	MATH 25600, 25900, or an approved alternative	3	three-quarter sequence in a minor field outside mathematics
14		17	

† *Credit may be granted by examination.*

* *May not include CMSC 10000, 10100, 10200, 11000, 11100, 11200, or any PHSC course lower than PHSC 18100.*

Degree Program in Applied Mathematics. Candidates for the B.S. in applied mathematics all take prescribed courses in numerical analysis, algebra, complex variables, ordinary differential equations, and partial differential equations. In addition, candidates are required to select, in consultation with the departmental counselor, a minor field, which consists of a three-course sequence that is outside the Department of Mathematics but within the Physical Sciences Collegiate Division.

Summary of Requirements:
Applied Mathematics

General Education		CHEM 11101-11201/11102-11202 or equivalent†, or PHYS12100-12200 or higher† MATH 13100-13200, 15100-15200, or 16100-16200†
Concentration	1	CHEM 11301/11302 or equivalent †, or PHYS 12300 or higher†
	1	MATH 13300, 15300, or 16300†
	3	MATH 20300-20400-20500 or 20700-20800-20900
	1	MATH 21100
	2	MATH 25400-25500 or 25700-25800
	3	MATH 27000-27300-27500
	6	courses within the PSCD but outside of mathematics, at least three of which should be taken in a single department*
	17	

† *Credit may be granted by examination.*
* *See restrictions on certain courses listed under previous summary.*

Degree Program in Mathematics with Specialization in Computer Science. The concentration program leading to a B.S. in mathematics with a specialization in computer science is a version of the B.S. in mathematics. The degree is in mathematics with the designation "with specialization in computer science" included on the final transcript. Candidates are required to complete a yearlong sequence in calculus (MATH 15100-15200-15300 or 16100-16200-16300 strongly recommended, but MATH 13100-13200-13300 may be used), in analysis (MATH 20300-20400-20500 or 20700-20800-20900), and in abstract algebra (MATH 25400-25500-25600 or 25700-25800-25900), and earn a grade of at least *C*- in each course. The remaining two mathematics courses may be chosen from the list of approved courses in the section Degree Programs in Mathematics except for MATH 17500 and STAT 25100; students are urged to take at least one of MATH 24200, 26200, 27700, or 28400. A *C* average or higher must be earned in these two courses.

Besides the third quarter of basic chemistry or basic physics, the seven courses required outside the Department of Mathematics must all be in the computer science department. A two-quarter sequence in programming is required; CMSC 15100-15200 is recommended. Five additional courses must be selected from among CMSC 10700 (only if CMSC 10600 was taken instead of CMSC 15200), CMSC 15300, CMSC 15400, or any computer science courses numbered 20000 or higher, except 27400. Students may substitute CMSC 10500-10600 instead of CMSC 15100-15200, but then should include CMSC 10700 as one of their additional five courses. Students must earn a grade of *C* or higher in each course taken in computer science to be eligible for this degree. For details, see the Computer Science section of this catalog.

Summary of Requirements:
Mathematics with Specialization in Computer Science

General Education		CHEM 11101-11201/11102-11202 or equivalent†, or PHYS12100-12200 or higher†
		MATH 13100-13200, 15100-15200, or 16100-16200†
Concentration	1	CHEM 11301/11302 or equivalent †, or PHYS 12300 or higher†
	1	MATH 13300, 15300, or 16300†
	3	MATH 20300-20400-20500 or 20700-20800-20900
	3	MATH 25400-25500-25600 or 25700-25800-25900
	2	CMSC 15100-15200
	2	approved courses in mathematics
	5	approved courses in computer science
	17	

† *Credit may be granted by examination.*

Degree Program in Mathematics with Specialization in Economics. This concentration program is a version of the B.S. in mathematics. The B.S. degree is in mathematics with the designation "with specialization in economics" included on the final transcript. Candidates are required to complete a yearlong sequence in calculus, in analysis (MATH 20300-20400-20500 or 20700-20800-20900), and two quarters of abstract algebra (MATH 25400-25500 or 25700-25800), and earn a grade of at least *C*- in each course. Students must also take STAT 25100 (Probability). The remaining two mathematics courses must include MATH 27000 (Complex Variables) and either MATH 27200 (Functional Analysis) for those interested in Econometrics or MATH 27300 (Ordinary Differential Equations) for those interested in economic theory. A *C* average or higher must be earned in these two courses.

Besides the third quarter of basic chemistry or basic physics, the eight courses required outside the Department of Mathematics must include STAT 22000 or 24400. The remaining seven courses should be in the economics department and must include ECON 20000-20100-20200-20300 and ECON 20900 or 21000 (Econometrics). The remaining two courses may be chosen from any undergraduate economics course numbered higher than ECON 20300. Students must earn a grade of *C* or higher in each course taken in economics to be eligible for this degree.

It is recommended that students considering graduate work in economics use some of their electives to include at least one programming course (CMSC 15100 is strongly recommended), and an additional course in statistics (STAT 24400-24500 is an appropriate two-quarter sequence). Students planning to apply to graduate economics programs are strongly encouraged to meet with one of the economics undergraduate program directors before the beginning of their third year.

Summary of Requirements:
Mathematics with Specialization in Economics

General Education		CHEM 11101-11201/11102-11202 or equivalent†, or PHYS12100-12200 or higher†
		MATH 13100-13200, 15100-15200, or 16100-16200†
Concentration	1	CHEM 11301/11302 or equivalent†, or PHYS 12300 or higher†
	1	MATH 13300, 15300, or 16300†
	3	MATH 20300-20400-20500 or 20700-20800-20900
	2	MATH 25400-25500 or 25700-25800
	1	MATH 27000
	1	MATH 27200 or 27300
	1	STAT 25100
	1	STAT 22000 or 24400
	4	ECON 20000-20100-20200-20300
	1	ECON 20900 or 21000
	2	courses in economics numbered higher than 20300
	18	

† *Credit may be granted by examination.*

Grading. Subject to College and concentration requirements and with consent of instructor, students *(except concentrators in mathematics or applied mathematics)* may register for quality grades, *P/N* grades, or *P/F* grades in any course beyond the second quarter of calculus. A *Pass* grade is given only for work of *C*- or higher.

Concentrators in mathematics or applied mathematics may take any 20000-level mathematics courses elected beyond concentration requirements for a grade of *P*. However, a grade of *C*- or higher must be earned in *each* calculus, analysis, or algebra course, and an overall grade average of *C* or higher must be earned in the remaining mathematics courses that a student uses to meet concentration requirements. PSCD courses taken to meet concentration requirements in mathematics must be taken for quality grades.

Incompletes are given in the Department of Mathematics only to those students who have done some work of passing quality and who are unable to complete all the course work by the end of the quarter. Arrangements are made between the instructor and the student.

Honors. The B.A. or B.S. with honors is awarded to students who, in addition to meeting requirements for one of the mathematics degrees, meet the following requirements: (1) a GPA of 3.25 or higher in mathematics courses and a 3.0 or higher overall; (2) completion of at least one honors sequence (either MATH 20700-20800-20900 or MATH 25700-25800-25900) with grades of *B*- or higher in each quarter; and (3) completion with a grade of *B*- or higher of at least five mathematics courses beyond requirement (2), chosen from the list that follows so that at least one course comes from each group (algebra, analysis, and topology).

Algebra courses: MATH 24100, 24200, 24300, 25700, 25800, 25900, 27700, 27800, 28400, 32500, 32600, 32700

Analysis courses: MATH 20700, 20800, 20900, 27000, 27200, 27300, 27400, 27500, 31200, 31300, 31400, 32100, 32200, 32300

Topology courses: MATH 26200, 26300, 31700, 31800, 31900

As approved, MATH 29700 (Proseminar in Mathematics) may be chosen so that it falls in any of the three groups. If both honors sequences are taken, one may be used for requirement (2) and one may be used in requirement (3). Courses taken for the honors requirements may be selected from courses taken to fulfill concentration requirements. Students interested in the honors degree should consult with the departmental counselor no later than the third quarter of their third year.

Joint Degree Program

B.A. (B.S.)/M.S. in Mathematics. Qualified College students may receive both a bachelor’s and a master’s degree in mathematics concurrently at the end of their years in the College. Qualification consists of satisfying all the requirements of each degree in mathematics. With the help of placement tests and honors sequences, able students can be engaged in 30000-level mathematics as early as their third year and, through an appropriate use of free electives, can complete the master's requirements by the end of their fourth year. To be eligible for this joint program, a student must begin in MATH 20700 in the Autumn Quarter of entrance. While only a few students complete the joint B.A./M.S. program, many undergraduates enroll in graduate-level mathematics courses. Admission to mathematics graduate courses requires prior written consent of the director or associate director of undergraduate studies. Interested students should apply to the departmental counselor as soon as possible but no later than the Winter Quarter of the third year.

Faculty

M. Abert, P. Achar, J. Alperin, D. Arinkin, L. Babai, W. Baily, Jr., A. Beilinson, S. Bloch, J. Borger, J. Brock, F. Cattaneo, P. Constantin, K. Corlette, J. Cowan, L. De Marco, V. Drinfeld, T. Dupont, A. Eskin, B. Farb, R. Fefferman, D. Gaitsgory, V. Ginzburg, G. Glauberman, P. Gordon, J. Grodal, D. Herrmann, D. Hirschfeldt, S. Hollander, C. Hruska, L. Kadanoff, C. Kenig, E.-W. Kirr, K. Koenig, R. Kottwitz, N. Lebovitz, M. Lewicka, A. Liulevicius, M. Mandell, J. P. May, N. Monod, R. Muchnik, M. Murthy, N. Nadirashvili, D. Nadler, R. Narasimhan, M. Nori, N. Nygaard, R. Pollack, M. Rothenberg, L. Ryzhik, P. Sally, Jr., L. R. Scott, R. Soare, S. Venkataramani, V. Vologodsky, S. Webster, S. Weinberger, A. Zuk

Courses: Mathematics (MATH)

10500-10600. Fundamental Mathematics I, II. *PQ: Adequate performance on the mathematics placement test. Students may not receive grades of* P/N *or* P/F *in this sequence. Students who place into this course must take it as first-year students. These courses do not meet the general education requirement in mathematical sciences. Both precalculus courses together count as only one elective.* This two-course sequence covers basic precalculus topics. The Autumn Quarter course is concerned with elements of algebra, coordinate geometry, and elementary functions. The Winter Quarter course continues with algebraic, circular, and exponential functions. *Autumn, Winter.*

11200-11300. Studies in Mathematics I, II. *PQ: MATH 10600, or placement into MATH 13100 or higher. Either of these course meets the general education requirement in mathematical sciences.* This sequence covers the basic conceptual foundations of mathematics by examining the ideas of number and symmetry. The first quarter addresses number theory, including a study of the rules of arithmetic, integral domains, primes and divisibility, congruences, and modular arithmetic. The second quarter's main topic is symmetry and geometry, including a study of polygons, Euclidean construction, polyhedra, group theory, and topology. The courses emphasize the understanding of ideas and the ability to express them through mathematical arguments. The courses are at the level of difficulty of the MATH 13100-13200-13300 calculus sequence. *11200-11300: Autumn, Winter. 11200: Spring.*

13100-13200-13300. Elementary Functions and Calculus I, II, III. *PQ: Invitation only based on appropriate performance on the mathematics placement test or MATH 10600. Students may not receive grades of* P/N *or* P/F *in the first two quarters of this sequence. MATH 13100-13200 meets the general education requirement in mathematical sciences.* This sequence provides the opportunity for students who are somewhat deficient in their precalculus preparation to complete the necessary background *and* cover basic calculus in three quarters. This is achieved through three regular one-hour class meetings and two mandatory one-and-one-half hour tutorial sessions each week. A class is divided into tutorial groups of about eight students each, and these meet with an undergraduate junior tutor for problem solving related to the course. The Autumn Quarter component of this sequence covers real numbers (algebraic and order properties), coordinate geometry of the plane (circles and lines), and real functions, and introduces the derivative. Topics examined in the Winter Quarter include differentiation, applications of the definite integral and the fundamental theorem. In the Spring Quarter, subjects include exponential and logarithmic functions, trigonometric functions, more applications of the definite integral, and Taylor expansions. Students are expected to understand the definitions of key concepts (limit, derivative, and integral) and to be able to apply definitions and theorems to solve problems. In particular, all calculus courses require students to do proofs. Students completing MATH 13100-13200-13300 have a command of calculus equivalent to that obtained in MATH 15100-15200-15300. *MATH 13300 is only offered in the Spring Quarter. Autumn, Winter, Spring.*

15100-15200-15300. Calculus I, II, III. *PQ: Superior performance on the mathematics placement test, or MATH 10600. Students may not receive grades of* P/N *or* P/F *in the first two quarters of this sequence. MATH 15100-15200 meets the general education requirement in mathematical sciences.* This is the regular calculus sequence in the department. Students entering this sequence are to have mastered appropriate precalculus material and, in many cases, have had some previous experience with calculus in high school or elsewhere. MATH 15100 undertakes a careful treatment of limits, the differentiation of algebraic and transcendental functions, and applications. Work in MATH 15200 is concerned with integration and additional techniques of integration. MATH 15300 deals with techniques and theoretical considerations, infinite series, and Taylor expansions. *MATH 15100 is offered only in the Autumn Quarter. Autumn, Winter, Spring.*

16100-16200-16300. Honors Calculus I, II, III. *PQ: Invitation only based on an outstanding performance on the calculus placement test. Students may not receive grades of* P/N *or* P/F *in the first two quarters of this sequence. MATH 16100-16200 meets the general education requirement in mathematical sciences.* MATH 16100-16200-16300 is an honors version of MATH 15100-15200-15300. A student with a strong background in the problem-solving aspects of one-variable calculus may, by suitable achievement on the calculus placement test, be permitted to register for MATH 16100-16200-16300. This sequence emphasizes the theoretical aspects of one-variable analysis and, in particular, the consequences of completeness in the real number system. *Autumn, Winter, Spring.*

17500. Elementary Number Theory. *PQ: Two quarters of calculus.* This course covers basic properties of the integers following from the division algorithm, primes and their distribution, congruences, existence of primitive roots, arithmetic functions, quadratic reciprocity, and other topics. Some transcendental numbers are covered. The subject is developed in a leisurely fashion, with many explicit examples. *Autumn.*

19510-19610. Mathematical Methods for Biological or Social Sciences I, II. *PQ: MATH 15300 or equivalent.* This sequence includes some linear algebra and three-dimensional geometry, a review of one-variable calculus, ordinary differential equations, partial derivatives, multiple integrals, partial differential equations, sequences, and series. *Summer; Autumn, Winter; Winter, Spring.*

20000-20100-20200. Mathematical Methods for Physical Sciences I, II, III. *PQ: MATH 15300 or equivalent. Entering students who have placement for MATH 15100-15200 may begin MATH 20000; such students have the requirement for MATH 15300 waived, but do not receive placement for MATH 15300.* This sequence is designed for students intending to major in the physical sciences (other than mathematics). MATH 20000 covers linear algebra and multivariable calculus. Topics include linear systems of equations, vector spaces, matrices, eigenvalue problems, partial derivatives, minimum and maximum problems, coordinate transformations, and multiple integrals. MATH 20100 deals with vector differential calculus, line integrals, theorems of Green, Gauss, and Stokes, complex numbers, introduction to ordinary differential equations, Fourier series, and partial differential equations. MATH 20200 is concerned with functions of a complex variable, Laplace and Fourier transforms, and an introduction to tensor calculus. *MATH 20000-20100-20200: Autumn, Winter, Spring; MATH 20000-20100: Winter, Spring.*

20300-20400-20500. Analysis in R^n I, II, III. *PQ: MATH 13300 or 15300 or 16300.* This three-course sequence is for students who intend to concentrate in mathematics or who require a rigorous treatment of analysis in several dimensions. Here, both the theoretical and problem-solving aspects of multivariable calculus are treated carefully. An introduction to metric spaces may also be included. Topics covered in MATH 20300 include the topology of R^n, compact sets, the geometry of Euclidean space, limits and continuous mappings, and partial differentiation. MATH 20400 deals with vector-valued functions, extrema, the inverse and implicit function theorems, and multiple integrals. MATH 20500 is concerned with line and surface integrals, and the theorems of Green, Gauss, and Stokes. One section of this course is intended for students who have taken MATH 13300 or who had a substandard performance in MATH 15300. This sequence is the basis for all advanced courses in analysis and topology. *Autumn, Winter, Spring; Winter, Spring, Autumn.*

20700-20800-20900. Honors Analysis in R^n I, II, III. *PQ: Invitation only.* This highly theoretical sequence in analysis is reserved for the most able students. The sequence covers the real number system, metric spaces, basic functional analysis, the Lebesgue integral, and other topics. *Autumn, Winter, Spring.*

21100. Basic Numerical Analysis. *PQ: MATH 20000 or 20300.* This course covers direct and iterative methods of solution of linear algebraic equations and eigenvalue problems. Topics include numerical differentiation and quadrature for functions of a single variable; approximation by polynomials and piece-wise polynomial functions; approximate solution of ordinary differential equations; and solution of nonlinear equations. *Spring.*

22000. Introduction to Mathematical Methods in Physics. *PQ: MATH 15200 or 16200, and PHYS 13200. This course is required of prospective physics concentrators with concurrent enrollment in PHYS 13300.* Topics include infinite and power series, complex numbers, linear equations and matrices, partial differentiation, multiple integrals, vector analysis, and Fourier series. Applications of these methods include Maxwell's equations, wave packets, and coupled oscillators. *Spring.*

22100. Mathematical Methods in Physics. *PQ: MATH 22000 and PHYS 13300, or PHYS 14200 and 14300. Required of physics concentrators.* Topics include ordinary and partial differential equations, calculus of variations, coordinate transformations, series solutions and orthogonal functions, integral transforms, and elements of complex analysis. *Autumn.*

24100. Topics in Geometry. *PQ: MATH 25500.* This course focuses on the interplay between abstract algebra (group theory, linear algebra, and the like) and geometry. Possible topics include affine geometry, projective geometry, bilinear forms, orthogonal geometry, and symplectic geometry. *Not offered 2003-04; will be offered 2004-05.*

24200. Algebraic Number Theory. *PQ: MATH 25500.* Factorization in Dedekind domains, integers in a number field, prime factorization, basic properties of ramification, and local degree are covered. *Spring.*

24300. Introduction to Algebraic Curves. *PQ: MATH 25500 or 25900, or consent of instructor.* This course covers the projective line and plane curves, both affine and projective. We also study conics and cubics, as well as the group law on the cubic. Abstract curves associated to function fields of one variable are discussed, along with the genus of a curve and the Riemann-Roch theorem. Curves of low genus are emphasized. *Not offered 2003-04; will be offered 2004-05.*

25000. Elementary Linear Algebra. *PQ: MATH 15200 or equivalent.* This course takes a concrete approach to the subject and includes some applications in the physical and social sciences. Topics include the theory of vector spaces and linear transformations, matrices and determinants, and characteristic roots and similarity. *Autumn, Spring.*

25400-25500-25600. Basic Algebra I, II, III. *PQ: MATH 13300 or 15300.* This sequence covers groups, subgroups, and permutation groups; rings and ideals; some work on fields; vector spaces, linear transformations and matrices, and modules; and canonical forms of matrices, quadratic forms, and multilinear algebra. *MATH 25600 is offered only in Spring Quarter. Autumn, Winter, Spring; Winter, Spring (MATH 25400-25500).*

25700-25800-25900. Honors Basic Algebra I, II, III. *PQ: MATH 15300 or 16300.* This is an accelerated version of MATH 25400-25500-25600. Topics include the theory of finite groups, commutative and noncommutative ring theory, modules, linear and multilinear algebra, and quadratic forms. The course also covers basic field theory, the structure of p-adic fields, and Galois theory. *Autumn, Winter, Spring.*

26100. Set Theory and Metric Spaces. *PQ: MATH 25400, or 20300 and 25000.* This course covers sets, relations, and functions; partially ordered sets; cardinal numbers; Zorn's lemma, well-ordering, and the axiom of choice; metric spaces; and completeness, compactness, and separability. *Autumn.*

26200. Point-Set Topology. *PQ: MATH 20300 and 25400.* This course examines topology on the real line, topological spaces, connected spaces and compact spaces, identification spaces and cell complexes, and projective and other spaces. With MATH 27400, this course forms a foundation for all advanced courses in analysis, geometry, and topology. *Winter.*

26300. Introduction to Algebraic Topology. *PQ: MATH 26200.* Topics include the fundamental group of a space; Van Kampen's theorem; covering spaces and groups of covering transformation; existence of universal covering spaces built up out of cells; and theorems of Gauss, Brouwer, and Borsuk-Ulam. *Spring.*

26700. Introduction to Representation Theory of Finite Groups. *PQ: MATH 25900 or 25600.* Topics include group algebras and modules, semisimple algebras and the theorem of Maschke; characters, character tables, orthogonality relations and calculation; induced representations and characters. Applications to permutation groups and solvability of groups are also included. *Autumn.*

26800. Introduction to Commutative Algebra. *PQ: MATH 25900 or 25600.* Topics covered include basic definitions and properties of commutative rings and modules, Noetherian and Artinian modules, exact sequences, Hilbert basis theorem, tensor products, localizations of rings and modules, associated primes and primary decomposition, Artin-Rees Lemma, Krull Intersection Theorem, completions, dimension theory of Noetherian rings, integral extensions, normal domains, Dedekind domains, going up and going down theorems, dimension of finitely generated algebras over a field, Affine varieties, Hilbert Nullstellensatz, dimension of affine varieties, product of affine varieties, and the dimension of intersection of subvarieties. *Winter.*

27000. Basic Complex Variables. *PQ: MATH 20500.* Topics include complex numbers, elementary functions of a complex variable, complex integration, power series, residues, and conformal mapping. *Autumn, Spring.*

27200. Basic Functional Analysis. *PQ: MATH 20900, or 26100 and 27000.* Banach spaces, bounded linear operators, Hilbert spaces, construction of the Lebesgue integral, L^p-spaces, Fourier transforms, Plancherel's theorem for R^n, and spectral properties of bounded linear operators are some of the topics discussed. *Winter.*

27300. Basic Theory of Ordinary Differential Equations. *PQ: MATH 20200 or 22100 or 27000.* This course covers first-order equations and inequalities, Lipschitz condition and uniqueness, properties of linear equations, linear independence, Wronskians, variation-of-constants formula, equations with constant coefficients and Laplace transforms, analytic coefficients, solutions in series, regular singular points, existence theorems, theory of two-point value problem, and Green's functions. *Winter.*

27400. Introduction to Differentiable Manifolds and Integration on Manifolds. *PQ: MATH 26200.* Topics include exterior algebra, differentiable manifolds and their basic properties, differential forms, integration on manifolds, Stoke's theorem, DeRham's theorem, and Sard's theorem. With MATH 26200, this course forms a foundation for all advanced courses in analysis, geometry, and topology. *Spring.*

27500. Basic Theory of Partial Differential Equations. *PQ: MATH 27300.* This course covers classification of second-order equations in two variables, wave motion and Fourier series, heat flow and Fourier integral, Laplace's equation and complex variables, second-order equations in more than two variables, Laplace operators, spherical harmonics, and associated special functions of mathematical physics. *Spring.*

27700. Mathematical Logic I. (=CMSC 27700) *PQ: MATH 25400.* This course provides an introduction to mathematical logic. Topics include propositional and predicate logic and the syntactic notion of proof versus the semantic notion of truth, including soundness and completeness. We also discuss the Goedel completeness theorem, the compactness theorem, and applications of compactness to algebraic problems. *Autumn.*

27800. Mathematical Logic II. (=CMSC 27800) *PQ: MATH 27700 or equivalent.* Some of the topics examined are number theory, Peano arithmetic, Turing compatibility, unsolvable problems, Gödel's incompleteness theorem, undecidable theories (e.g., the theory of groups), quantifier elimination, and decidable theories (e.g., the theory of algebraically closed fields). *Winter.*

27900. Logic and Logic Programming. (=CMSC 21500) *PQ: MATH 25400 or CMSC 27700, or consent of instructor. Students are encouraged to take both CMSC 21500 and 27700. Programming knowledge not required.* Predicate logic is a precise logical system developed to formally express mathematical reasoning. Prolog is a computer language intended to implement a portion of predicate logic. This course covers both predicate logic and Prolog, which are presented to complement each other and to illustrate the principles of logic programming and automated theorem proving. Topics include syntax and semantics of propositional and predicate logic, tableaux proofs, resolution, Skolemization, Herbrand's theorem, unification, and refining resolution. It includes weekly classes and programming assignments in Prolog (e.g., searching, backtracking, cut). *R. Soare. Winter.*

28000. Introduction to Formal Languages. (=CMSC 28000) *PQ: MATH 25000 or 25500, and experience with mathematical proofs.* Topics include automata theory, regular languages, CFLs, and Turing machines. This course is a basic introduction to computability theory and formal languages. *J. Simon. Autumn.*

28100. Introduction to Complexity Theory. (=CMSC 28100) *PQ: MATH 25000 or 25500 or CMSC 27100, and experience with mathematical proofs.* Computability topics are discussed (e.g., the s-m-n theorem and the recursion theorem, resource-bounded computation). This course introduces complexity theory. Relationships between space and time, determinism and nondeterminism, NP-completeness, and the P versus NP question are investigated. *E. Vigoda. Spring*

28400. Honors Combinatorics and Probability. (=CMSC 27400) *PQ: MATH 25000 or 25400, or CMSC 27100, or consent of instructor. Experience with mathematical proofs.* Methods of enumeration, construction, and proof of existence of discrete structures are discussed in conjunction with the basic concepts of probability theory over a finite sample space. Enumeration techniques are applied to the calculation of probabilities, and, conversely, probabilistic arguments are used in the analysis of combinatorial structures. Among other topics are basic counting, linear recurrences, generating functions, Latin squares, finite projective planes, graph theory, Ramsey theory, coloring graphs and set systems, random variables, independence, expected value, standard deviation, and Chebyshev's and Chernoff's inequalities. *L. Babai. Spring.*

29700. Proseminar in Mathematics. *PQ: General education mathematics sequence. Consent of instructor and departmental counselor. Open to mathematics concentrators only. Students are required to submit the College Reading and Research Course Form. Must be taken for a quality grade. Autumn, Winter, Spring.*

30000-30100. Set Theory I, II. *PQ: Consent of instructor.* MATH 30000 is a course on axiomatic set theory with applications to the undecidability of mathematical statements. Topics include axioms of Zermelo-Fraenkel (ZF) set theory; Von Neumann rank and reflection principles; the Levy hierarchy and absoluteness; inner models; Gödel's Constructible sets (L), the consistency of the Axiom of Choice (AC), and the Generalized Continuum Hypothesis (GCH); and Souslin's Hypothesis in L. MATH 30100 deals with models of set theory coding of syntax; Cohen's method of forcing and the unprovability of AC and GCH; Martin's axiom and the unprovability of Souslin's Hypothesis; Solovay's model in which every set of reals is Lebesgue Measurable; inaccessible and measurable cardinals; and analytic determinateness, Silver indiscernibles for L (O-Sharp), larger cardinals (elementary embeddings and compactness), and the axiom of determinateness. *Not offered 2003-04; will be offered 2004-05.*

30200. Computability Theory I. (=CMSC 38000) *PQ: MATH 25500 or consent of instructor.* MATH 30200 begins with models for defining computable functions such as the recursive functions and those computable by a Turing machine. Topics include the Kleene normal form theorem for representing computable functions and computably enumerable (c.e.) sets; the enumeration and s-m-n theorem, unsolvable problems, classification of c.e. sets, the Kleene arithmetic hierarchy, coding of information from one set to another, various degrees for measuring noncomputability, many-one, truth-table, and Turing degrees. The course also includes the Kleene recursion theorem and its applications, other fixed point theorems such as the Arslanov completeness criterion, elementary properties of Turing degrees, generic sets, and the construction of various non-c.e. degrees by oracle Kleene-Post constructions. *R. Soare. Winter.*

30300. Computability Theory II. (=CMSC 38100) *PQ: MATH 30200.* MATH 30300 develops the deeper properties of computability and the classification of relative computability on sets and (Turing) degrees. It begins with the finite injury priority method of Friedberg and Muchnik. It continues with the infinite injury priority method of Sacks and the minimal pair of computably enumerable (c.e) degrees method by Lachlan. It introduces the tree method of Lachlan for classifying more difficult priority constructions, and it works out many properties of the c.e. degrees and the algebraic structure of the c.e. sets. It presents results on the relationship between a c.e. set and the degree of information it encodes such as the high maximal set theorem of Martin. *R. Soare. Spring.*

30500. Computability and Complexity Theory. (=CMSC 38500) *PQ: Consent of instructor.* Part one consists of models for defining computable functions, such as primitive recursive functions, (general) recursive functions, and Turing machines; and their equivalence, the Church-Turing Thesis, unsolvable problems, diagonalization, and properties of computably enumerable (c.e.) sets. Part two deals with Kolmogorov complexity (resource bounded complexity) that studies the quantity of information in individual objects and uses the book by Li and Vitanyi. The third part covers functions computable with special bounds on time and space of the Turing machine, such as polynomial time computability, the classes P and NP, nondeterministic Turing machines, NP-complete problems, polynomial time hierarchy, and P-space complete problems. *Autumn. Not offered 2003-04; will be offered 2004-05.*

30900-31000. Model Theory I, II. *PQ: MATH 25500.* MATH 30900 covers completeness and compactness; elimination of quantifiers; omission of types; elementary chains and homogeneous models; two cardinal theorems by Vaught, Chang, and Keisler; categories and functors; inverse systems of compact Hausdorf spaces; and applications of model theory to algebra. In MATH 31000, we study saturated models; categoricity in power; the Cantor-Bendixson and Morley derivatives; the Morley theorem and the Baldwin-Lachlan theorem on categoricity; rank in model theory; uniqueness of prime models and existence of saturated models; indiscernibles; ultraproducts; and differential fields of characteristic zero. *Not offered 2003-04; will be offered 2004-05.*

31200-31300-31400. Analysis I, II, III. *PQ: MATH 26200, 27000, 27200, and 27400; and consent of director or associate director of undergraduate studies.* Topics include Lebesgue measure, abstract measure theory, and Riesz representation theorem; basic functional analysis (L^p-spaces, elementary Hilbert space theory, Hahn-Banach, open mapping theorem, and uniform boundedness); Radon-Nikodym theorem, duality for L^p-spaces, Fubini's theorem, differentiation, Fourier transforms, locally convex spaces, weak topologies, and convexity; compact operators; spectral theorem and integral operators; Banach algebras and general spectral theory; Sobolev spaces and embedding theorems; Haar measure; and Peter-Weyl theorem, holomorphic functions, Cauchy's theorem, harmonic functions, maximum modulus principle, meromorphic functions, conformal mapping, and analytic continuation. *Autumn, Winter, Spring.*

31700-31800-31900. Topology and Geometry I, II, III. *PQ: MATH 26200, 26300, 27000, 27200, and 27400; and consent of director or associate director of undergraduate studies.* MATH 31700 covers smooth manifolds, tangent bundles, vector fields, Frobenius theorem, Sard's theorem, Whitney embedding theorem, and transversality. MATH 31800 considers fundamental group and covering spaces; Lie groups and Lie algebras; and principal bundles, connections, introduction to Riemannian geometry, geodesics, and curvature. Topics in MATH 31900 are cell complexes, homology, and cohomology; and Mayer-Vietoris theorem, Kunneth theorem, cup products, duality, and geometric applications. *Autumn, Winter, Spring.*

32000-32100-32200. Mathematical and Statistical Methods for the Neurosciences I, II, III. (MATH 32100=STAT 24700/31000) *PQ: Equivalent of one year of college calculus required. Prior course in linear algebra (e.g., MATH 25000) required. Prior course in differential equations (e.g., MATH 27300) recommended. At least one course in neurobiology (e.g., BIOS 14106 or 24236, or NURB 31800) recommended.* This three-quarter sequence is for students interested in computational and theoretical neuroscience. It introduces various mathematical and statistical ideas and techniques used in the analysis of brain mechanisms. The first quarter introduces mathematical ideas and techniques in a neuroscience context. Topics include some coverage of matrices and complex variables; eigenvalue problems, spectral methods, and Greens functions for differential equations; and some discussion of both deterministic and probabilistic modeling in the neurosciences. The second quarter treats statistical methods that are important in understanding nervous system function. It includes basic concepts of mathematical probability; and information theory, discrete Markov processes, and time series. The third quarter covers more advanced topics that include perturbation and bifurcation methods for the study of dynamical systems, symmetry, methods, and some group theory. A variety of applications to neuroscience are described. *Autumn, Winter, Spring.*

32500-32600-32700. Algebra I, II, III. *PQ: MATH 25700-25800-25900, and consent of director or associate director of undergraduate studies.* MATH 32500 deals with groups and commutative rings. MATH 32600 investigates elements of the theory of fields and of Galois theory, as well as noncommutative rings. MATH 32700 introduces other basic topics in algebra. *Autumn, Winter, Spring.*