

# Mathematics

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## Program of Study

The Department of Mathematics provides an environment of research and comprehensive instruction in mathematics and applied mathematics at both undergraduate and graduate levels. Both a B.A. and a B.S. program in mathematics are offered, including a B.S. degree in applied mathematics and a B.S. degree in mathematics with a specialization in economics. Students in other fields of study may also complete a minor in mathematics; information follows the description of the major.

The requirements for a degree in mathematics or in applied mathematics express the educational intent of the Department of Mathematics; they are drawn with an eye toward the cumulative character of an education based in mathematics, the present emerging state of mathematics, and the scholarly and professional prerequisites of an academic career in mathematics.

Requirements for each bachelor's degree look to the advancement of students' general education in modern mathematics and their knowledge of its relation with the other sciences (B.S.) or with the other arts (B.A.).

Descriptions of the detailed requirements that give meaning to these educational intentions follow. Students should understand that any particular degree requirement can be modified if persuasive reasons are presented to the department; petitions to modify requirements are submitted in person to the director of undergraduate studies or to one of the departmental counselors.

**Placement.** At what level does an entering student begin mathematics at the University of Chicago? This question is answered individually for students on the basis of their performance on one of two placement tests in mathematics administered during Orientation in September: either a precalculus mathematics placement test or a calculus placement test. Scores on the *mathematics* placement test determine the appropriate beginning mathematics course for each student: a

precalculus course (MATH 10500) or one of three other courses (MATH 11200, 13100, or 15100). Students who wish to begin at a level higher than MATH 15100 *must* take the *calculus* placement test, unless they receive Advanced Placement credit as described in the following paragraphs.

Students who submit a score of 5 on the AB Advanced Placement exam in mathematics or a score of 4 on the BC Advanced Placement exam in mathematics receive credit for MATH 15100. Students who submit a score of 5 on the BC Advanced Placement exam in mathematics receive credit for MATH 15100 and 15200. Currently no course credit is offered in the Mathematics Department at Chicago for work done in an International Baccalaureate Program or for British A-level or O-level examinations.

Students with suitable achievement on the calculus placement test are invited to begin with Honors Calculus (MATH 16100) or beyond. Excellent scores on the calculus placement test may give placement credit for one, two, or three quarters of calculus. The strong recommendation from the department is that students who have AP credit for one or two quarters of calculus enroll in Honors Calculus (MATH 16100) when they enter as first-year students. This course builds on the strong computational background provided in AP courses and best prepares entering students for further study in mathematics.

Admission to Honors Analysis (MATH 20700) is by invitation only to those first-year students with superior performance on the calculus placement test or to those sophomores who receive a strong recommendation from their instructor in MATH 16100-16200-16300. Students who are granted three quarters of calculus credit on the basis of the calculus placement test and who do not qualify for admission to Honors Analysis (MATH 20700) will place into MATH 19900 (Introduction to Analysis and Linear Algebra). These students should consult with one of the departmental counselors about the option of beginning with Honors Calculus (MATH 16100) so that they would be eligible for admission to Honors Analysis the following year.

## Program Requirements

**Undergraduate Programs.** Four bachelor's degrees are available in the Department of Mathematics: the B.A. in mathematics, the B.S. in mathematics, the B.S. in applied mathematics, and the B.S. in mathematics with specialization in economics. Programs qualifying students for the degree of B.A. provide more elective freedom, while programs qualifying students for the degrees of B.S. require more emphasis in the physical sciences. All degree programs, whether qualifying students for a degree in mathematics or in applied mathematics, require fulfillment of the College's general education requirements. The general education sequence in the physical sciences must be selected from either first-year basic chemistry or first-year basic physics. The major includes at least nine courses in mathematics (detailed descriptions follow for each degree), plus at least five courses within the Physical Sciences Collegiate Division (PSCD) but

outside mathematics, one of which completes the three-quarter sequence in basic chemistry or basic physics, and at least two others which should be from a single department. These latter courses must be chosen from astronomy, chemistry, computer science, physics (12000s or above), geophysical sciences, or statistics (22000 or above). We particularly call attention to the degree program that is described in more detail in the following paragraph regarding the B.S. in mathematics with specialization in economics.

NOTE: Students who are majoring in mathematics may use AP credit for chemistry and/or physics to meet their general education physical sciences requirement and/or the physical sciences component of the major. However, no credit designated simply as “physical science,” from AP examinations or from the College’s physical sciences placement or accreditation examination, may be used in their general education requirement or in the mathematics major.

Students who are majoring in mathematics are required to complete: a 10000-level sequence in calculus (or to demonstrate equivalent competence on the calculus placement test); a three-quarter sequence in analysis (MATH 20300-20400-20500 or 20700-20800-20900); and two quarters of an algebra sequence (MATH 25400-25500 or 25700-25800). MATH 25700-25800-25900 is designated as an honors version of Basic Algebra. Registration for this course is the option of the individual student. Consultation with one of the departmental counselors is strongly advised. The normal procedure is to take calculus in the first year, analysis in the second, and algebra in the third.

**Degree Programs in Mathematics.** Candidates for the B.A. and B.S. in mathematics take a sequence in basic algebra. B.A. candidates may opt for a two-quarter sequence (MATH 25400-25500 or 25700-25800), whereas candidates for the B.S. degree must take the three-quarter algebra sequence (MATH 25400-25500-25600 or 25700-25800-25900).

The remaining mathematics courses needed in the programs (three for the B.A., two for the B.S.) must be selected, with due regard for prerequisites, from the following approved list of mathematics courses. STAT 25100 also meets the requirement. B.A. candidates may include MATH 25600 or 25900. Mathematics courses in the Paris Spring Mathematics Program may also be used to meet this requirement, and one of these three courses will be accepted as a replacement for MATH 25600 or 25900 for B.S. candidates. Students may not use both MATH 16300 and 19900 to meet major or minor requirements.

### *List of Approved Courses*

17500	19900	21100	24100	24200	24300	26100	26200
26300	26700	26800	27000	27200	27300	27400	27500
27700	27800	27900	28000	28100	28400	29200	29700*
30000	30100	30200	30300	30900	31000	31200	31300
31400	31700	31800	31900	32500	32600	32700	

\* as approved

B.S. candidates are further required to select a minor field, which consists of three additional courses that are outside the Department of Mathematics and either are within the same department in the Physical Sciences Collegiate Division or are among the Computational Neuroscience courses in the Biological Sciences Collegiate Division. These courses must be chosen in consultation with one of the departmental counselors.

**Summary of Requirements: Mathematics**

<i>General Education</i>		CHEM 11101-11201/11102-11202 or equivalent*, or PHYS12100-12200 or higher* MATH 13100-13200, 15100-15200, or 16100-16200*
<i>Major</i>	1	CHEM 11301/11302 or equivalent*, or PHYS 12300 or higher*
	1	MATH 16300* or 19900**
	3	MATH 20300-20400-20500 or 20700-20800-20900
	2	mathematics courses chosen from Approved List
	4	courses within the PSCD but outside of mathematics, at least two of which should be taken in a single department***

*plus the following requirements:*

	B.A.		B.S.	
	2	MATH 25400-25500 or 25700-25800	3	MATH 25400-25500-25600 or 25700-25800-25900
	1	MATH 25600, 25900, or an approved alternative	3	courses that are not MATH courses but are either from the same PSCD department or are CPNS courses
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\* Credit may be granted by examination.

\*\* Students who complete (or receive credit for) MATH 13300 or 15300 may use these courses as general electives, but 19900 must be completed for the major.

\*\*\* May include BIOS 24221-24222-24223. May not include CMSC 10000, 10100, 10200, 11000, 11100, or 11200, or any PHSC course lower than PHSC 18100.

**Degree Program in Applied Mathematics.** Candidates for the B.S. in applied mathematics all take prescribed courses in numerical analysis, algebra, complex variables, ordinary differential equations, and partial differential equations. In addition, candidates are required to select, in consultation with one of the departmental counselors, a secondary field, which consists of three additional

courses from a single department that is outside the Department of Mathematics but within the Physical Sciences Collegiate Division, or among the Computational Neuroscience courses in the Biological Sciences Collegiate Division.

### Summary of Requirements: Applied Mathematics

<i>General Education</i>		CHEM 11101-11201/11102-11202 or equivalent*, or PHYS 12100-12200 or higher* MATH 13100-13200, 15100-15200, or 16100-16200*
<i>Major</i>	1	CHEM 11301/11302 or equivalent*, or PHYS 12300 or higher*
	1	MATH 16300* or 19900
	3	MATH 20300-20400-20500 or 20700-20800-20900
	1	MATH 21100
	2	MATH 25400-25500 or 25700-25800
	3	MATH 27000-27300-27500
	6	courses that are not MATH courses but are either within the PSCD or are CPNS courses, at least three of which should be taken in a single department**

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\* *Credit may be granted by examination.*

\*\* *See restrictions on certain courses listed under previous summary.*

### Degree Program in Mathematics with Specialization in Economics.

This program is a version of the B.S. in mathematics. The B.S. degree is in mathematics with the designation “with specialization in economics” included on the final transcript. Candidates are required to complete a yearlong sequence in calculus, in analysis (MATH 20300-20400-20500 or 20700-20800-20900), and two quarters of abstract algebra (MATH 25400-25500 or 25700-25800), and earn a grade of at least *C-* in each course. Students must also take STAT 25100 (Probability). The remaining two mathematics courses must include MATH 27000 (Complex Variables) and either MATH 27200 (Functional Analysis) for those interested in Econometrics or MATH 27300 (Ordinary Differential Equations) for those interested in economic theory. A *C* average or higher must be earned in these two courses.

Besides the third quarter of basic chemistry or basic physics, the eight courses required outside the Department of Mathematics must include STAT 23400 or 24400. The remaining seven courses should be in the economics department and must include ECON 20000-20100-20200-20300 and ECON 20900 or 21000 (Econometrics). The remaining two courses may be chosen from any undergraduate economics course numbered higher than ECON 20300. Students

must earn a grade of *C* or higher in each course taken in economics to be eligible for this degree.

It is recommended that students considering graduate work in economics use some of their electives to include at least one programming course (CMSC 15100 is strongly recommended), and an additional course in statistics (STAT 24400-24500 is an appropriate two-quarter sequence). Students planning to apply to graduate economics programs are strongly encouraged to meet with one of the economics undergraduate program directors before the beginning of their third year.

### Summary of Requirements: Mathematics with Specialization in Economics

<i>General Education</i>		CHEM 11101-11201/11102-11202 or equivalent*, or PHYS 12100-12200 or higher* MATH 13100-13200, 15100-15200, or 16100-16200*
<i>Major</i>	1	CHEM 11301/11302 or equivalent*, or PHYS 12300 or higher*
	1	MATH 16300* or 19900
	3	MATH 20300-20400-20500 or 20700-20800-20900
	2	MATH 25400-25500 or 25700-25800
	1	MATH 27000
	1	MATH 27200 or 27300
	1	STAT 25100
	1	STAT 23400 or 24400
	4	ECON 20000-20100-20200-20300
	1	ECON 20900 or 21000
	<u>2</u>	courses in economics numbered higher than 20300
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\* *Credit may be granted by examination.*

**Grading.** Subject to College grading requirements and grading requirements for the major and with consent of instructor, students (*except students who are majoring in mathematics or applied mathematics*) may take any course beyond the second quarter of calculus for either a quality grade or for *P/F* grading. A *Pass* grade is given only for work of *C-* or higher.

Mathematics or applied mathematics students may take any 20000-level mathematics courses elected beyond program requirements on a *P/F* basis. However, a grade of *C-* or higher must be earned in *each* calculus, analysis, or algebra course, and an overall grade average of *C* or higher must be earned in the remaining mathematics courses that a student uses to meet requirements for the major. All courses taken to meet requirements in the mathematics major must be taken for quality grades.

Incompletes are given in the Department of Mathematics only to those students who have done some work of passing quality and who are unable to complete all the course work by the end of the quarter. Arrangements are made between the instructor and the student.

**Honors.** The B.A. or B.S. with honors is awarded to students who, while meeting requirements for one of the mathematics degrees, also meet the following requirements: (1) a GPA of 3.25 or higher in mathematics courses and a 3.0 or higher overall; (2) no grade below *C*- and no grade of *W* in any mathematics course; (3) completion of at least one honors sequence (either MATH 20700-20800-20900 or 25700-25800-25900) with grades of *B*- or higher in each quarter; and (4) completion with a grade of *B*- or higher of at least five mathematics courses chosen from the list that follows so that at least one course comes from each group (i.e., algebra, analysis, and topology). If both honors sequences are taken, one sequence may be used for requirement (3) and one sequence may be used for up to three of the five courses in requirement (4).

Algebra courses: MATH 24100, 24200, 24300, 25700, 25800, 25900, 26700, 26800, 27700, 27800, 28400, 32500, 32600, 32700

Analysis courses: MATH 20700, 20800, 20900, 27000, 27200, 27300, 27400, 27500, 31200, 31300, 31400, 32100, 32200, 32300

Topology courses: MATH 26200, 26300, 31700, 31800, 31900

As approved, MATH 29700 (Proseminar in Mathematics), or any course(s) in the Paris Spring Mathematics Program, may be chosen so that it falls in any of the three groups. One of the three Paris courses each year will be accepted as a replacement for MATH 25900 for candidates who are working toward graduation with honors. Courses taken for the honors requirements (3) and (4) also may be counted toward courses taken to meet requirements for the major. Students who wish to be considered for honors should consult with one of the departmental counselors no later than Spring Quarter of their third year.

### Minor Program in Mathematics

The minor in mathematics requires a total of six or seven courses in mathematics, depending on whether or not the third quarter of calculus is required in another degree program. If it is not used elsewhere, the third quarter of calculus (MATH 13300 or 15300 or 16300, or placement credit) must be included in the minor, for a total of seven courses. The remaining six courses must include a three-course sequence in Analysis (MATH 20300-20400-20500 or 20700-20800-20900) and a two-course sequence in Algebra (MATH 25400-25500 or 25700-25800). The sixth course may be chosen from either the third quarter of Algebra (MATH 25600 or 25900) or a mathematics course numbered 24000 or higher chosen in consultation with the director of undergraduate studies or one of the departmental counselors. Under special circumstances and to avoid double counting, students

may also use mathematics courses numbered 24000 or higher to substitute for up to two quarters of Analysis or Algebra, if these are required in another degree program.

No course in the minor can be double counted with the student's major(s) or with other minors; nor can it be counted toward general education requirements. Students must earn a grade of at least C- in each of the courses in the mathematics minor. More than one-half of the requirements for a minor must be met by registering for courses bearing University of Chicago course numbers.

Students must meet with the director of undergraduate studies or one of the departmental counselors by Spring Quarter of their third year to declare their intention to complete a minor program in mathematics and to obtain approval for the minor on a form obtained from their College adviser. Courses for the minor are chosen in consultation with the director of undergraduate studies or one of the departmental counselors.

## Joint Degree Program

**B.A. (B.S.)/M.S. in Mathematics.** Qualified College students may receive both a bachelor's and a master's degree in mathematics concurrently at the end of their studies in the College. Qualification consists of satisfying all requirements of each degree in mathematics. With the help of placement tests and honors sequences, able students can be engaged in 30000-level mathematics as early as their third year and, through an appropriate use of free electives, can complete the master's requirements by the end of their fourth year. To be eligible for the joint program, a student must begin MATH 20700 in Autumn Quarter of entrance. While only a few students complete the joint B.A./M.S. program, many undergraduates enroll in graduate-level mathematics courses. Admission to mathematics graduate courses requires prior written consent of the director or co-director of undergraduate studies. Students should submit their application for the joint program to one of the departmental counselors as soon as possible, but no later than the Winter Quarter of their third year.

## Faculty

M. Abert, J. Alperin, V. Angeltveit, L. Babai, W. Baily, Jr., M. Bainbridge, T. Basak, A. Beilinson, D. Biss, S. Bloch, J. Boller, D. Boyarchenko, N. Broaddus, P. Constantin, K. Corlette, J. Cowan, L. De Marco, V. Drinfeld, J. Dubédat, T. Dupont, M. Ershov, A. Eskin, B. Farb, R. Fefferman, T. Fiore, F. Gancedo, V. Ginzburg, G. Glauberger, G. Grigoriy, J. Grodal, R. Hadani, D. Herrmann, D. Hirschfeldt, L. Kadanoff, C. Kenig, M. Kerr, D. Kim, Y.-S. Kim, M. Kisin, C. Klivans, G. Koch, R. Kottwitz, G. Lawler, N. Lebovitz, R. Lee, A. Liulevicius, J. P. May, D. McReynolds, A. Montalban, M. Murthy, R. Narasimhan, M. Nori, N. Nygaard, M. Popa, M. Rothenberg, L. Ryzhik, P. Sally, Jr., W. Schlag, B. Schmidt, L. R. Scott, P. Seidel, R. Soare, J. Souto, C. Tsogka, V. Vologodsky, N. Wahl, S. Webster, S. Weinberger, A. Wienhard, S. Ziesler, A. Zlatos, R. Zimmer



## Courses: Mathematics (MATH)

**10500-10600. Fundamental Mathematics I, II.** *PQ: Adequate performance on the mathematics placement test. Must be taken for a quality grade. Students who place into this course must take it as first-year students. These courses do not meet the general education requirement in mathematical sciences. Both precalculus courses together count as only one elective.* This two-course sequence covers basic precalculus topics. The Autumn Quarter course is concerned with elements of algebra, coordinate geometry, and elementary functions. The Winter Quarter course continues with algebraic, trigonometric, and exponential functions. *Autumn, Winter.*

**11200-11300. Studies in Mathematics I, II.** *PQ: MATH 10600, or placement into MATH 13100 or higher. While students may take MATH 11300 without having taken 11200, it is strongly recommended that 11200 be taken first. Either course in this sequence meets the general education requirement in mathematical sciences.* These courses are at the level of difficulty of the MATH 13100-13200-13300 calculus sequence. This sequence covers the basic conceptual foundations of mathematics by examining the ideas of number and symmetry. The first quarter addresses number theory, including a study of the rules of arithmetic, integral domains, primes and divisibility, congruences, and modular arithmetic. The second quarter's main topic is symmetry and geometry, including a study of polygons, Euclidean construction, polyhedra, group theory, and topology. The courses emphasize the understanding of ideas and the ability to express them through mathematical arguments. *MATH 11200-11300: Autumn, Winter; MATH 11200: Spring.*

**13100-13200-13300. Elementary Functions and Calculus I, II, III.** *PQ: Invitation only based on adequate performance on the mathematics placement test or MATH 10600. Students may not take the first two quarters of this sequence for P/F grading. MATH 13100-13200 meets the general education requirement in mathematical sciences.* This sequence provides the opportunity for students who are somewhat deficient in their precalculus preparation to complete the necessary background *and* cover basic calculus in three quarters. This is achieved through three regular one-hour class meetings and two mandatory one-and-one-half hour tutorial sessions each week. A class is divided into tutorial groups of about eight students each, and these meet with an undergraduate junior tutor for problem solving related to the course. The Autumn Quarter component of this sequence gives a careful treatment of limits and the continuity and differentiability of algebraic functions. Topics examined in the Winter Quarter include applications of differentiation; exponential, logarithmic, and trigonometric functions; the definite integral and the fundamental theorem, and applications of the integral. In the Spring Quarter, subjects include more applications of the definite integral, infinite sequences and series, and Taylor expansions. Students are expected to understand the definitions of key concepts (i.e., limit, derivative, integral) and to be able to apply definitions and theorems to solve problems. In particular, all calculus courses require students to do proofs. Students completing MATH

13100-13200-13300 have a command of calculus equivalent to that obtained in 15100-15200-15300. *MATH 13100-13200-13300: Autumn, Winter, Spring; MATH 13100-13200: Winter, Spring.*

**15100-15200-15300. Calculus I, II, III.** *PQ: Superior performance on the mathematics placement test, or MATH 10600. Students may not take the first two quarters of this sequence for P/F grading. MATH 15100-15200 meets the general education requirement in mathematical sciences.* This is the regular calculus sequence in the department. Students entering this sequence are to have mastered appropriate precalculus material and, in many cases, have had some previous experience with calculus in high school or elsewhere. MATH 15100 undertakes a careful treatment of limits, the differentiation of algebraic and transcendental functions, and applications. Work in MATH 15200 is concerned with the mean value theorem, integration, techniques of integration, and applications of the integral. MATH 15300 deals with additional techniques and theoretical considerations of integration, infinite sequences and series, and Taylor expansions. All Autumn Quarter offerings of MATH 15100, 15200, and 15300 begin with a rigorous treatment of limits and limit proofs. *MATH 15100-15200-15300: Autumn, Winter, Spring; MATH 15200-15300: Autumn, Winter; MATH 15300: Autumn.*

**16100-16200-16300. Honors Calculus I, II, III.** *PQ: Invitation only based on superior performance on the calculus placement test. Students may not take the first two quarters of this sequence for P/F grading. MATH 16100-16200 meets the general education requirement in mathematical sciences.* MATH 16100-16200-16300 is an honors version of MATH 15100-15200-15300. A student with a strong background in the problem-solving aspects of one-variable calculus may, by suitable achievement on the calculus placement test, be permitted to register for MATH 16100-16200-16300. This sequence emphasizes the theoretical aspects of one-variable analysis and, in particular, the consequences of completeness in the real number system. MATH 16300 also includes an introduction to linear algebra. *Autumn, Winter, Spring.*

**17500. Elementary Number Theory.** *PQ: Two quarters of calculus.* This course covers basic properties of the integers following from the division algorithm, primes and their distribution, congruences, existence of primitive roots, arithmetic functions, quadratic reciprocity, and other topics. Some transcendental numbers are covered. The subject is developed in a leisurely fashion, with many explicit examples. *Autumn.*

**19520. Mathematical Methods for Social Sciences.** *PQ: MATH 15300 or equivalent.* This course takes a concrete approach to the basic topics of multivariable calculus. Topics include a brief review of one-variable calculus, parametric equations, alternate coordinate systems, vectors and vector functions, partial derivatives, multiple integrals, and Lagrange multipliers. *Autumn, Winter, Spring.*

**19620. Linear Algebra.** *PQ: MATH 13300 or 15200.* This course takes a concrete approach to the basic topics of linear algebra. Topics include vector geometry, systems of linear equations, vector spaces, matrices and determinants, and eigenvalue problems. *Autumn, Winter, Spring.*

**19900. Introduction to Analysis and Linear Algebra.** *PQ: Superior performance on the mathematics placement test or MATH 15300 or 13300. Students may not use both MATH 16300 and 19900 to satisfy major or minor requirements.* This course is intended for students who are making the transition from MATH 15300 to 20300, or for students who need more preparation in learning to read and write proofs. This course covers the fundamentals of theoretical mathematics and prepares students for upper-level mathematics courses beginning with MATH 20300. Topics include the construction of the real numbers, completeness and the least upper bound property, the topology of the real line, the structure of finite-dimensional vector spaces over the real and complex numbers. *Autumn, Winter, Spring.*

**20000-20100. Mathematical Methods for Physical Sciences I, II.** *PQ for MATH 20000: MATH 15300 or 19620 or equivalent. PQ for MATH 20100: MATH 20000, or both 19520 and 19620, or equivalent.* This sequence is intended for students who are majoring in a department in the Physical Sciences Collegiate Division other than mathematics. MATH 20000 covers multivariable calculus, including the algebra and geometry of Euclidean space, differentiation and integration of functions of several variables, vector valued functions and the classical theorems of vector analysis (i.e., theorems of Green, Gauss, and Stokes). MATH 20100 introduces ordinary differential equations (e.g., first and second order linear differential equations, series solutions, and the Laplace transform) and complex analysis (i.e., basic properties of the complex plane and analytic functions through Cauchy's theorem). *Autumn, Winter; Winter, Spring.*

**20300-20400-20500. Analysis in  $\mathbb{R}^n$  I, II, III.** *PQ: MATH 16300 or 19900.* This three-course sequence is intended for students who plan to major in mathematics or who require a rigorous treatment of analysis in several dimensions. Both theoretical and problem-solving aspects of multivariable calculus are treated carefully. Topics in MATH 20300 include metric spaces, the topology of  $\mathbb{R}^n$ , compact sets, the geometry of Euclidean space, limits and continuous mappings, and partial differentiation. MATH 20400 deals with vector-valued functions, extrema, the inverse and implicit function theorems, and multiple integrals. MATH 20500 is concerned with line and surface integrals; and the theorems of Green, Gauss, and Stokes. This sequence is the basis for all advanced courses in analysis and topology. *Autumn, Winter, Spring; Winter, Spring, Autumn; Spring, Autumn, Winter.*

**20700-20800-20900. Honors Analysis in  $\mathbb{R}^n$  I, II, III.** *PQ: Invitation only.* This highly theoretical sequence in analysis is intended for the most able students. We cover the real number system, metric spaces, basic functional analysis, the Lebesgue integral, and other topics. *Autumn, Winter, Spring.*

**21100. Basic Numerical Analysis.** *PQ: MATH 20000 or 20300.* This course covers direct and iterative methods of solution of linear algebraic equations and eigenvalue problems. Topics include numerical differentiation and quadrature for functions of a single variable, approximation by polynomials and piece-wise polynomial functions, approximate solution of ordinary differential equations, and solution of nonlinear equations. *Spring.*

**22000. Introduction to Mathematical Methods in Physics.** *PQ: MATH 15200 or 16200, and PHYS 13200.* This course, with concurrent enrollment in *PHYS 13300*, is required of students who plan to major in physics. Topics include infinite series and power series, complex numbers, linear equations and matrices, partial differentiation, multiple integrals, vector analysis, and Fourier series. Applications of these methods include Maxwell's equations, wave packets, and coupled oscillators. *Spring.*

**24100. Topics in Geometry.** *PQ: MATH 25500 or 25800.* This course focuses on the interplay between abstract algebra (group theory, linear algebra, and the like) and geometry. Several of the following topics are covered: affine geometry, projective geometry, bilinear forms, orthogonal geometry, and symplectic geometry. *This course is offered in alternate years. Spring.*

**24200. Algebraic Number Theory.** *PQ: MATH 25500 or 25800.* Topics include factorization in Dedekind domains, integers in a number field, prime factorization, basic properties of ramification, and local degree. *Spring. Offered 2008–09; not offered 2007–08.*

**24300. Introduction to Algebraic Curves.** *PQ: MATH 25500 or 25800, or consent of instructor. MATH 25600 or 25900 is strongly recommended.* This course covers the projective line and plane curves, both affine and projective. We also study conics and cubics, as well as the group law on the cubic. Abstract curves associated to function fields of one variable are discussed, along with the genus of a curve and the Riemann-Roch theorem. Curves of low genus are emphasized. *Spring.*

**25400-25500-25600. Basic Algebra I, II, III.** *PQ: MATH 16300 or 19900.* This sequence covers groups, subgroups, and permutation groups; rings and ideals; some work on fields; vector spaces, linear transformations and matrices, and modules; and canonical forms of matrices, quadratic forms, and multilinear algebra. *MATH 25400-25500-25600: Autumn, Winter, Spring; MATH 25400-25500: Winter, Spring.*

**25700-25800-25900. Honors Basic Algebra I, II, III.** *PQ: MATH 16300 or 19900.* This sequence is an accelerated version of *MATH 25400-25500-25600*. Topics include the theory of finite groups, commutative and noncommutative ring theory, modules, linear and multilinear algebra, and quadratic forms. The course also covers basic field theory, the structure of  $p$ -adic fields, and Galois theory. *Autumn, Winter, Spring.*

**26100. Set Theory and Metric Spaces.** *PQ: MATH 25400 or 20300.* This course covers sets, relations, and functions; partially ordered sets; cardinal numbers; Zorn's lemma, well-ordering, and the axiom of choice; metric spaces; and completeness, compactness, and separability. *Autumn.*

**26200. Point-Set Topology.** *PQ: MATH 20300 or 20700, and 25400 or 25700.* This course examines topology on the real line, topological spaces, connected spaces and compact spaces, identification spaces and cell complexes, and projective and other spaces. With MATH 27400, it forms a foundation for all advanced courses in analysis, geometry, and topology. *Winter.*

**26300. Introduction to Algebraic Topology.** *PQ: MATH 26200.* Topics include the fundamental group of a space; Van Kampen's theorem; covering spaces and groups of covering transformation; existence of universal covering spaces built up out of cells; and theorems of Gauss, Brouwer, and Borsuk-Ulam. *Spring.*

**26700. Introduction to Representation Theory of Finite Groups.** *PQ: MATH 25900 or 25600.* Topics include group algebras and modules, semisimple algebras and the theorem of Maschke; characters, character tables, orthogonality relations and calculation; and induced representations and characters. Applications to permutation groups and solvability of groups are also included. *Autumn.*

**26800. Introduction to Commutative Algebra.** *PQ: MATH 25900 or 25600.* Topics include basic definitions and properties of commutative rings and modules, Noetherian and Artinian modules, exact sequences, Hilbert basis theorem, tensor products, localizations of rings and modules, associated primes and primary decomposition, Artin-Rees Lemma, Krull intersection theorem, completions, dimension theory of Noetherian rings, integral extensions, normal domains, Dedekind domains, going up and going down theorems, dimension of finitely generated algebras over a field, Affine varieties, Hilbert Nullstellensatz, dimension of affine varieties, product of affine varieties, and the dimension of intersection of subvarieties. *Winter.*

**27000. Basic Complex Variables.** *PQ: MATH 20500 or 20800.* Topics include complex numbers, elementary functions of a complex variable, complex integration, power series, residues, and conformal mapping. *Autumn, Spring.*

**27200. Basic Functional Analysis.** *PQ: MATH 20900 or 27000.* Topics include Banach spaces, bounded linear operators, Hilbert spaces, construction of the Lebesgue integral,  $L^p$ -spaces, Fourier transforms, Plancherel's theorem for  $\mathbb{R}^n$ , and spectral properties of bounded linear operators. *Winter.*

**27300. Basic Theory of Ordinary Differential Equations.** *PQ: MATH 27000 or PHYS 22100.* This course covers first-order equations and inequalities, Lipschitz condition and uniqueness, properties of linear equations, linear independence, Wronskians, variation-of-constants formula, equations with constant coefficients

and Laplace transforms, analytic coefficients, solutions in series, regular singular points, existence theorems, theory of two-point value problem, and Green's functions. *Winter.*

**27400. Introduction to Differentiable Manifolds and Integration on Manifolds.** *PQ: MATH 26200.* Topics include exterior algebra; differentiable manifolds and their basic properties; differential forms; integration on manifolds; and the theorems of Stoke, DeRham, and Sard. With MATH 26200, this course forms a foundation for all advanced courses in analysis, geometry, and topology. *Spring.*

**27500. Basic Theory of Partial Differential Equations.** *PQ: MATH 27300.* This course covers classification of second-order equations in two variables, wave motion and Fourier series, heat flow and Fourier integral, Laplace's equation and complex variables, second-order equations in more than two variables, Laplace operators, spherical harmonics, and associated special functions of mathematical physics. *Spring.*

**27700. Mathematical Logic I.** (=CMSC 27700) *PQ: MATH 25400 or 25700.* This course introduces mathematical logic. Topics include propositional and predicate logic and the syntactic notion of proof versus the semantic notion of truth (e.g., soundness, completeness). We also discuss the Goedel completeness theorem, the compactness theorem, and applications of compactness to algebraic problems. *A. Montalban. Autumn.*

**27800. Mathematical Logic II.** (=CMSC 27800) *PQ: MATH 27700 or equivalent.* Some topics examined are number theory, Peano arithmetic, Turing compatibility, unsolvable problems, Gödel's incompleteness theorem, undecidable theories (e.g., the theory of groups), quantifier elimination, and decidable theories (e.g., the theory of algebraically closed fields). *A. Montalban. Winter.*

**27900. Logic and Logic Programming.** (=CMSC 21500) *PQ: MATH 25400 or CMSC 27700, or consent of instructor. Students are encouraged to take both CMSC 21500 and 27700. Programming knowledge not required.* Predicate logic is a precise logical system developed to formally express mathematical reasoning. Prolog is a computer language intended to implement a portion of predicate logic. This course covers both predicate logic and Prolog, which are presented to complement each other and to illustrate the principles of logic programming and automated theorem proving. Topics include syntax and semantics of propositional and predicate logic, tableaux proofs, resolution, Skolemization, Herbrand's theorem, unification, and refining resolution. Students are required to attend weekly classes and to complete programming assignments in Prolog (e.g., searching, backtracking, cut). *R. Soare, W. Sterner. Winter. Offered 2008–09; not offered 2007–08.*

**28000. Introduction to Formal Languages.** (=CMSC 28000) PQ: CMSC 15300, or MATH 25000 or 25500. This course is a basic introduction to computability theory and formal languages. Topics include automata theory, regular languages, context-free languages, and Turing machines. *J. Simon. Autumn.*

**28100. Introduction to Complexity Theory.** (=CMSC 28100) PQ: CMSC 27100, or MATH 25000 or 25500; and experience with mathematical proofs. Computability topics are discussed (e.g., the s-m-n theorem and the recursion theorem, resource-bounded computation). This course introduces complexity theory. Relationships between space and time, determinism and nondeterminism, NP-completeness, and the P versus NP question are investigated. *K. Mulmuley. Spring.*

**28400. Honors Combinatorics and Probability.** (=CMSC 27400) PQ: MATH 25000 or 25400, or CMSC 27100, or consent of instructor. Experience with mathematical proofs. Methods of enumeration, construction, and proof of existence of discrete structures are discussed in conjunction with the basic concepts of probability theory over a finite sample space. Enumeration techniques are applied to the calculation of probabilities, and, conversely, probabilistic arguments are used in the analysis of combinatorial structures. Other topics include basic counting, linear recurrences, generating functions, Latin squares, finite projective planes, graph theory, Ramsey theory, coloring graphs and set systems, random variables, independence, expected value, standard deviation, and Chebyshev's and Chernoff's inequalities. *This course is offered in alternate years. L. Babai. Spring.*

**29700. Proseminar in Mathematics.** PQ: General education mathematics sequence. Consent of instructor and departmental counselor. Open only to students who are majoring in mathematics. Students are required to submit the College Reading and Research Course Form. Must be taken for a quality grade. *Autumn, Winter, Spring.*

**30000-30100. Set Theory I, II.** PQ: Consent of instructor. MATH 30000 is a course on axiomatic set theory with applications to the undecidability of mathematical statements. Topics include axioms of Zermelo-Fraenkel (ZF) set theory; Von Neumann rank and reflection principles; the Levy hierarchy and absoluteness; inner models; Gödel's Constructible sets (L), the consistency of the Axiom of Choice (AC), and the Generalized Continuum Hypothesis (GCH); and Souslin's Hypothesis in L. MATH 30100 deals with models of set theory coding of syntax; Cohen's method of forcing and the unprovability of AC and GCH; Martin's axiom and the unprovability of Souslin's Hypothesis; Solovay's model in which every set of reals is Lebesgue Measurable; inaccessible and measurable cardinals; and analytic determinateness, Silver indiscernibles for L (O-Sharp), larger cardinals (elementary embeddings and compactness), and the axiom of determinateness. *This course is offered in alternate years. Winter, Spring.*



**30200-30300. Computability Theory I, II.** (=CMSC 38000-38100) *PQ: MATH 25500 or consent of instructor.* CMSC 38000 is concerned with recursive (computable) functions and sets generated by an algorithm (recursively enumerable sets). Topics include various mathematical models for computations (e.g., Turing machines and Kleene schemata, enumeration and s-m-n theorems, the recursion theorem, classification of unsolvable problems, priority methods for the construction of recursively enumerable sets and degrees). CMSC 38100 treats classification of sets by the degree of information they encode, algebraic structure and degrees of recursively enumerable sets, advanced priority methods, and generalized recursion theory. *This course is offered in alternate years. R. Soare. Winter, Spring.*

**30500-30600. Computability and Complexity Theory I, II.** (=CMSC 38500) *PQ: Consent of instructor.* Part one consists of models for defining computable functions: primitive recursive functions, (general) recursive functions, and Turing machines; the Church-Turing Thesis; unsolvable problems; diagonalization; and properties of computably enumerable sets. Part two deals with Kolmogorov (resource bounded) complexity: the quantity of information in individual objects. Part three covers functions computable with time and space bounds of the Turing machine: polynomial time computability, the classes P and NP, NP-complete problems, polynomial time hierarchy, and P-space complete problems. *L. Fortnow. Autumn. 2008–09; not offered 2007–08.*

**30900-31000. Model Theory I, II.** *PQ: MATH 25500 or 25800.* MATH 30900 covers completeness and compactness; elimination of quantifiers; omission of types; elementary chains and homogeneous models; two cardinal theorems by Vaught, Chang, and Keisler; categories and functors; inverse systems of compact Hausdorff spaces; and applications of model theory to algebra. In MATH 31000, we study saturated models; categoricity in power; the Cantor-Bendixson and Morley derivatives; the Morley theorem and the Baldwin-Lachlan theorem on categoricity; rank in model theory; uniqueness of prime models and existence of saturated models; indiscernibles; ultraproducts; and differential fields of characteristic zero. *Offered 2008–09; not offered 2007–08.*

**31200-31300-31400. Analysis I, II, III.** *PQ: MATH 26200, 27000, 27200, and 27400; and consent of director or co-director of undergraduate studies.* Topics include Lebesgue measure, abstract measure theory, and Riesz representation theorem; basic functional analysis ( $L^p$ -spaces, elementary Hilbert space theory, Hahn-Banach, open mapping theorem, and uniform boundedness); Radon-Nikodym theorem, duality for  $L^p$ -spaces, Fubini's theorem, differentiation, Fourier transforms, locally convex spaces, weak topologies, and convexity; compact operators; spectral theorem and integral operators; Banach algebras and general spectral theory; Sobolev spaces and embedding theorems; Haar measure; and Peter-Weyl theorem, holomorphic functions, Cauchy's theorem, harmonic functions, maximum modulus principle, meromorphic functions, conformal mapping, and analytic continuation. *Autumn, Winter, Spring.*



**31700-31800-31900. Topology and Geometry I, II, III.** *PQ: MATH 26200, 26300, 27000, 27200, and 27400; and consent of director or co-director of undergraduate studies.* MATH 31700 covers smooth manifolds, tangent bundles, vector fields, Frobenius theorem, Sard's theorem, Whitney embedding theorem, and transversality. MATH 31800 considers fundamental group and covering spaces; Lie groups and Lie algebras; and principal bundles, connections, introduction to Riemannian geometry, geodesics, and curvature. Topics in MATH 31900 are cell complexes, homology, and cohomology; and Mayer-Vietoris theorem, Kunneth theorem, cup products, duality, and geometric applications. *Autumn, Winter, Spring.*

**32000-32100-32200. Mathematical and Statistical Methods for the Neurosciences I, II, III.** *PQ: Equivalent of one year of college calculus required. Prior course in linear algebra (e.g., MATH 25000) required. Prior course in differential equations (e.g., MATH 27300) recommended. At least one course in neurobiology (e.g., BIOS 14106 or 24236, or NURB 31800) recommended.* This three-quarter sequence is intended for students who are interested in computational and theoretical neuroscience. It introduces various mathematical and statistical ideas and techniques used in the analysis of brain mechanisms. The first quarter introduces mathematical ideas and techniques in a neuroscience context. Topics include some coverage of matrices and complex variables; eigenvalue problems, spectral methods, and Greens functions for differential equations; and some discussion of both deterministic and probabilistic modeling in the neurosciences. The second quarter treats statistical methods that are important in understanding nervous system function. It includes basic concepts of mathematical probability; and information theory, discrete Markov processes, and time series. The third quarter covers more advanced topics that include perturbation and bifurcation methods for the study of dynamical systems, symmetry, methods, and some group theory. A variety of applications to neuroscience are described. *Autumn, Winter, Spring.*

**32500-32600-32700. Algebra I, II, III.** *PQ: MATH 25700-25800-25900, and consent of director or co-director of undergraduate studies.* MATH 32500 deals with groups and commutative rings. MATH 32600 investigates elements of the theory of fields and of Galois theory, as well as noncommutative rings. MATH 32700 introduces other basic topics in algebra. *Autumn, Winter, Spring.*

**37500. Algorithms in Finite Groups.** (=CMSC 36500) *PQ: Linear algebra, finite fields, and a first course in group theory (Jordan-Holder and Sylow theorem) required. Prior knowledge of algorithms not required; there will be no programming problems.* We consider the asymptotic complexity of some of the basic problems of computational group theory. The course demonstrates the relevance of a mix of mathematical techniques, ranging from combinatorial ideas, the elements of probability theory, and elementary group theory, to the theories of rapidly mixing Markov chains, applications of simply stated consequences of the Classification of Finite Simple Groups (CFSG), and, occasionally, detailed information about finite simple groups. *L. Babai. Spring. Offered 2008–09; not offered 2007–08.*