Solutions to Homework 5

February 18, 2015

Exercise 1 (Ex 6.3.2, page 251)**.** *Convert the grammar*

$$\begin{aligned} S &\rightarrow aAA \\ A &\rightarrow aS \mid bS \mid a \end{aligned}$$

to a PDA that accepts the same language by empty stack.

Proof. Let $M = (\{q\}, \{a, b\}, \{A, S\}, \delta, q, S, \emptyset)$ be a PDA defined by

- $\delta(q, a, S) = \{(q, AA)\}$
- $\delta(q, a, A) = \{(q, \epsilon), (q, S)\}$
- $\delta(q, b, A) = \{(q, S)\}$

□

Exercise 2 (Ex 6.3.5, page 252)**.** *Below are some context-free languages. For each, devise a PDA that accepts the language by empty stack. You may, if you wish, first construct a grammar for the language, and then convert to a PDA.*

a) $\{a^n b^m c^{2(n+m)} : n \geq 0, m \geq 0\}$.

b) $\{a^i b^j c^k : i = 2j \text{ or } j = 2k\}$.

c) $\{0^n 1^m : n \leq m \leq 2n\}$.

Proof. a) Let $M = (\{q_1, q_2, q_3\}, \{a, b, c\}, \{S, z_0\}, \delta, q_1, z_0, \emptyset)$ be a PDA defined by

- $\delta(q_1, a, A) = \{(q_1, SSA)\}$ for all $A \in \Gamma$.
- $\delta(q_1, \epsilon, A) = \{(q_2, A)\}$ for all $A \in \Gamma$.

- $\delta(q_2, b, A) = \{(q_2, SSA)\}$.
- $\delta(q_2, \epsilon, A) = \{(q_3, A)\}$ for all $A \in \Gamma$.
- $\delta(q_3, c, S) = \{(q_2, \epsilon)\}$.
- $\delta(q_3, \epsilon, z_0) = \{(q_3, \epsilon)\}$.

b) Following is a CFG generating the language.

$$\begin{array}{rcl} S & \rightarrow & PC \mid AQ \\ P & \rightarrow & aaPb \mid \epsilon \\ C & \rightarrow & cC \mid \epsilon \\ A & \rightarrow & aA \mid \epsilon \\ Q & \rightarrow & bbQc \mid \epsilon \end{array}$$

Let $M = (\{q\}, \{a, b, c\}, \{a, b, c, A, B, P, Q, S\}, \delta, q, S, \emptyset)$, where

- $\delta(q, \epsilon, S) = \{(q, PC), (q, AQ)\}$.
- $\delta(q, \epsilon, P) = \{(q, aaPb), (q, \epsilon)\}$.
- $\delta(q, \epsilon, C) = \{(q, cC), (q, \epsilon)\}$.
- $\delta(q, \epsilon, A) = \{(q, aA), (q, \epsilon)\}$.
- $\delta(q, \epsilon, Q) = \{(q, bbQc), (q, \epsilon)\}$.
- $\delta(q, a, a) = \{(q, \epsilon)\}$.
- $\delta(q, b, b) = \{(q, \epsilon)\}$.
- $\delta(q, c, c) = \{(q, \epsilon)\}$.

c) Following grammar generates the language.

$$S \rightarrow 0S1 \mid 0S11 \mid \epsilon$$

Let $M = (\{q\}, \{0, 1\}, \{0, 1, S\}, \delta, q, S, \emptyset)$, where

- $\delta(q, \epsilon, S) = \{(q, 0S1), (q, 0S11), (q, \epsilon)\}$.
- $\delta(q, 0, 0) = \{(q, \epsilon)\}$.
- $\delta(q, 1, 1) = \{(q, \epsilon)\}$.

□

Exercise 3 (Ex 7.2.2, page 286)**.** *When we try to apply the pumping lemma to a CFL, the "adversary wins," and we cannot complete the proof. Show what goes wrong when we choose L to be one of the following languages:*

a) $\{00, 11\}$.

b) $\{0^n 1^n : n \geq 1\}$.

c) The set of palindromes over alphabet $\{0, 1\}$.

Proof. a) Let n be the constant as in the pumping lemma. If $n > 2$, then pumping lemma is trivially true.

b) Let $z = 0^m 1^m \in L$, where $|z| > n$. According to pumping lemma, $z = uvwxy$. If $v = 1$, and $x = 0$, then $uv^iwx^iy \in L$ for every $i \geq 0$, and thus z can be pumped.

c) A palindrome is either of the form $z = \beta\beta^R$ or $z = \beta a \beta^R$, where $a \in \Sigma$ and $\beta \in \Sigma^*$. According to pumping lemma, $z = uvwxy$. In either case, let v be the *last* symbol in β, and x be the *first* symbol in β^R, then $uv^iwx^iy \in L$ for every $i \geq 0$.

□

Exercises are from the book "Automata Theory, Language, and Computation", 3rd edition, by John E. Hopcroft, Rajeev Motwani, and Jeffrey D. Ullman, published by Addison-Wesley.