CMSC 28100-1 Spring 2016
Homework 3

April 21, 2016

1. (HMU 8.4.1) Informally but clearly describe multitape Turing machines that accept each of the following languages from Exercise 8.2.2 (HW2, problem 1). Try to make each of your Turing machines run in time proportional to the input length.

(a) The set of strings with an equal number of 0's and 1's.

(b) $\{a^n b^n c^n \mid n \geqslant 1\}$

(c) $\{ww^R \mid w$ is any string of 0's and 1's$\}$, where w^R is the reverse of a string. For instance, $10010^R = 01001$.

2. Design (single tape) Turing machines for the following languages.

(a) $L = \{w \in \{0,1\}^* : w$ contains more zeroes than ones $\}$.

(b) $L = \{w \in \{0,1\}^* : w$ contains twice as many zeroes as ones $\}$.

3. (HMU 8.3.2) A common operation in Turing machine programs involves "shifting over". Ideally, we would like to create an extra cell at the current head position, in which we could store some character. However, we cannot edit the tape in this way. Rather, we need to move the contents of each of the cells to the right of the current head position one cell right, and then nd our way back to the current head position. Show how to perform this operation. Hint: Leave a special symbol to mark the position to which the head must return.

4. Give a sufficiently detailed description of how to construct a Turing machine which computes the exponentiation function $f(n) = 2^n$. You can assume the input n is given in unary form 0^n, and your Turing machine should produce a unary representation of the number 2^n (i.e., a sequence of 2^n consecutive 0s), and halt.