CMSC 22880 - Day 8

Looking Ahead

Today:

Static variables & methods in Java

Adding on to gates

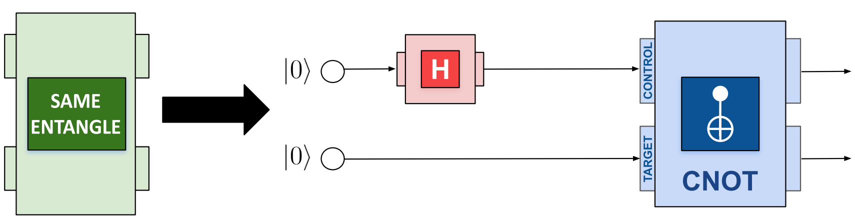
Working up to:

Superdense codes

Quantum teleportation

Midterm: February 8th, in-person during class

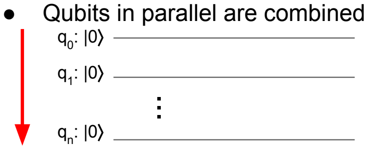
The Same Entangle Circuit



- How could we construct the the Opposite Entangle Circuit?
- What other types of entangled states can we create?
- What gates do we need if both qubits start in $|0\rangle$?

The Quantum Register

- Quantum Registers hold collection of qubits used for computation, $q_0, q_1, \ldots, q_n$
  - One or more may be used in a quantum circuit
  - Qubits are initialized to the ground state $|0\rangle$
- Within circuit diagrams, each qubit in the register is represented with a wire
- Qubits in parallel are combined to form an overall state with the ***tensor product***



$q_0 \otimes q_1 \otimes \ldots \otimes q_n = |0\rangle \otimes |0\rangle \otimes \ldots \otimes |0\rangle = |00\ldots0\rangle$

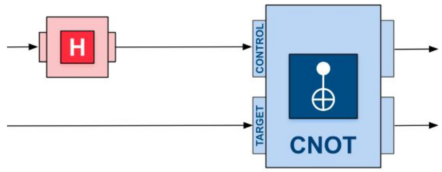
Remember: when combining qubits:

Top to Bottom

reads

Left to Right

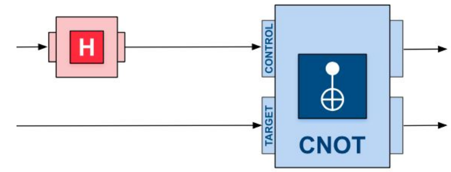
Combined Same Entangle Matrix



What is the equation to solve this?

a) M = (H⊗Identity) x CNOT
b) M = CNOT x (H⊗Identity)
c) M = (H x Identity) ⊗ CNOT
d) M = CNOT ⊗ (H x Identity)

Combined Same Entangle Matrix



M = CNOT(H⊗Identity)

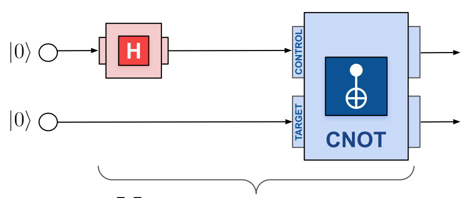
$$\boldsymbol{M} = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1&1\\1&-1\end{bmatrix}\otimes\begin{bmatrix}1&0\\0&1\end{bmatrix}\right)$$

$$\boldsymbol{M} = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1&0&1&0\\0&1&0&1\\1&0&-1&0\\0&1&0&-1\end{bmatrix}\right)$$

Same Entangle Circuit

$$\boldsymbol{M} = \frac{1}{\sqrt{2}}\begin{bmatrix}1&0&1&0\\0&1&0&1\\0&1&0&-1\\1&0&-1&0\end{bmatrix}$$

Calculating Output with $|00\rangle$



M = CNOT(H⊗Identity)

$|\psi_{out}\rangle = \boldsymbol{M}|\psi_{in}\rangle$

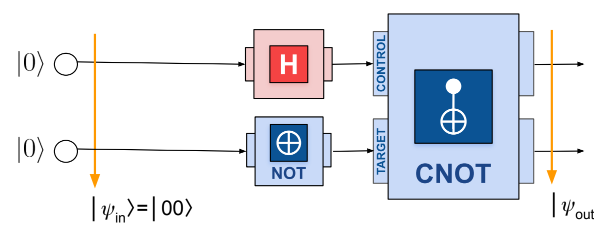
$|\psi_{out}\rangle =$ ***CNOT(H⊗Identity)***$|00\rangle$

$$|\psi_{out}\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1&0&1&0\\0&1&0&1\\0&1&0&-1\\1&0&-1&0\end{bmatrix}\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

$$|\psi_{out}\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\1\end{bmatrix} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Same entangle state results!

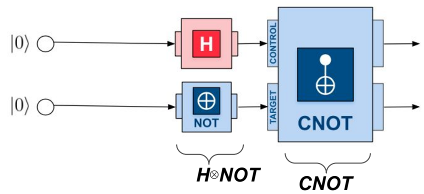
New Circuit: Add NOT gate



Additional NOT gate transforms bottom qubit to $|1\rangle$ before entering entangling circuit.

What do you think will happen to the output qubit state?

New Circuit: NOT Bottom Qubit



$|\psi_{out}\rangle = \boldsymbol{M}|\psi_{in}\rangle$

$|\psi_{out}\rangle = \boldsymbol{CNOT(H \otimes NOT)}|00\rangle$

$$|\psi_{out}\rangle = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1&1\\1&-1\end{bmatrix}\otimes\begin{bmatrix}0&1\\1&0\end{bmatrix}\right)\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

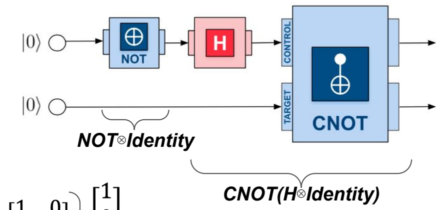
$$|\psi_{out}\rangle = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}\left(\frac{1}{\sqrt{2}}\begin{bmatrix}0&1&0&1\\1&0&1&0\\0&1&0&-1\\1&0&-1&0\end{bmatrix}\right)\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

$$|\psi_{out}\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}0&1&0&1\\1&0&1&0\\1&0&-1&0\\0&1&0&-1\end{bmatrix}\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

This circuit created the opposite entangle state!

$$|\psi_{out}\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

Adding NOT before H



$|\psi_{out}\rangle = \boldsymbol{M}|\psi_{in}\rangle$

$|\psi_{out}\rangle = \boldsymbol{CNOT(H \otimes Identity)(NOT \otimes Identity)}|00\rangle$

$$|\psi_{out}\rangle = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1&1\\1&-1\end{bmatrix}\otimes\begin{bmatrix}1&0\\0&1\end{bmatrix}\right)\left(\begin{bmatrix}0&1\\1&0\end{bmatrix}\otimes\begin{bmatrix}1&0\\0&1\end{bmatrix}\right)\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

$$|\psi_{out}\rangle = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1&0&1&0\\0&1&0&1\\1&0&-1&0\\0&1&0&-1\end{bmatrix}\begin{bmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{bmatrix}\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

$$|\psi_{out}\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1&0&1&0\\0&1&0&1\\0&-1&0&1\\-1&0&1&0\end{bmatrix}\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

We created same entangle state… WITH PHASE!

$$|\psi_{out}\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\-1\end{bmatrix} = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

Entangled Two Qubit States

Different circuit variations produce ***same/opposite*** entangle ***with/without*** phase

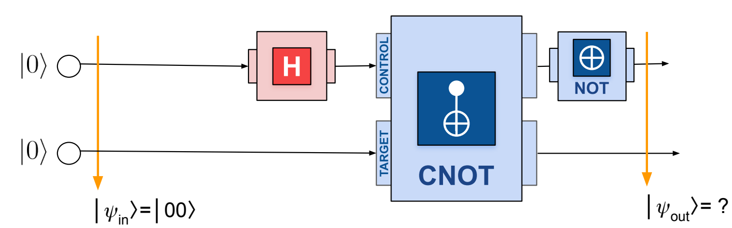
- We just looked at a common example that uses H+CNOT as foundation….many other entangling circuits exist!
- Resulting states are ***Bell States***

Bell States

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}, \quad |\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\1\\0\end{bmatrix}$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\-1\end{bmatrix}, \quad |\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\-1\\0\end{bmatrix}$$

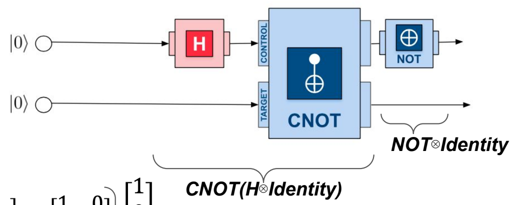
New Circuit: Add NOT gate after Entangle



Additional NOT gate on top qubit after entangle.

What do you think will happen to the output qubit state?

NOT after Entangle



$|\psi_{out}\rangle = \boldsymbol{M}|\psi_{in}\rangle$

$|\psi_{out}\rangle = \boldsymbol{(NOT{\otimes}Identity)CNOT(H{\otimes}Identity)}|00\rangle$

$$|\psi_{out}\rangle = \left(\begin{bmatrix}0&1\\1&0\end{bmatrix}\otimes\begin{bmatrix}1&0\\0&1\end{bmatrix}\right)\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1&1\\1&-1\end{bmatrix}\otimes\begin{bmatrix}1&0\\0&1\end{bmatrix}\right)\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

$$|\psi_{out}\rangle = \begin{bmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1&0&1&0\\0&1&0&1\\1&0&-1&0\\0&1&0&-1\end{bmatrix}\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

$$|\psi_{out}\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}0&1&0&-1\\1&0&-1&0\\1&0&1&0\\0&1&0&1\end{bmatrix}\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

This circuit also creates the opposite entangle state!

$$|\psi_{out}\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

PRACTICE: What state results when |01> is the input to this circuit?

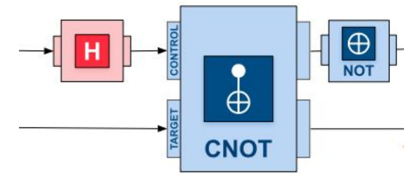
HINT: Matrix: $\frac{1}{\sqrt{2}}\begin{bmatrix}0&1&0&-1\\1&0&-1&0\\1&0&1&0\\0&1&0&1\end{bmatrix}$

A. $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

C. $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

B. $\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$

D. $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$



PRACTICE: What state results when |01> is the input to this circuit?

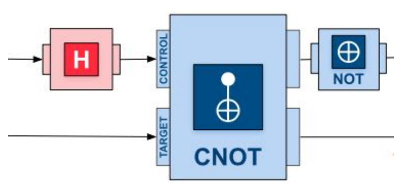
HINT: Matrix: $\frac{1}{\sqrt{2}}\begin{bmatrix}0&1&0&-1\\1&0&-1&0\\1&0&1&0\\0&1&0&1\end{bmatrix}$

A. $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

C. $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

B. $\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$

D. $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$



PRACTICE: Which matrix corresponds to circuit that produces two-qubit entangled states?

$$\begin{bmatrix}1&0&0&0\\0&0&0&1\\0&0&1&0\\0&1&0&0\end{bmatrix} \qquad \frac{1}{\sqrt{2}}\begin{bmatrix}1&0&1&0\\0&1&0&1\\0&-1&0&1\\-1&0&1&0\end{bmatrix}$$

A. The left

B. The right

C. Both

D. Neither

PRACTICE: Which matrix corresponds to circuit that produces two-qubit entangled states?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{bmatrix}$$

A. The left

B. The right

C. Both

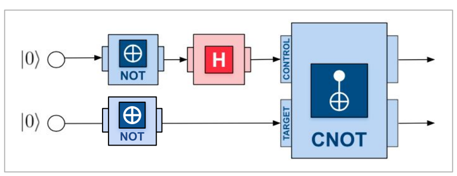
D. Neither

Maximally-entangled, two qubit states are often referred to as:

a. Feynman States
b. CNOT States
c. Bell States
d. Q - States

(True / False) The only way to entangle two qubits is with a CNOT and a H gate.

Determine the quantum state that results from the following circuit:



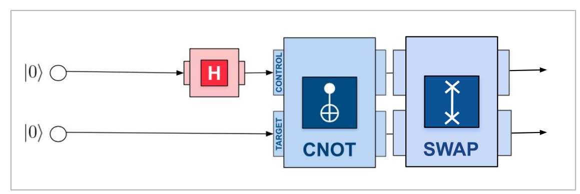
a. $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

b. $|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$

c. $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$

d. $|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}$

(True / False) The output qubits of this quantum circuit are entangled.

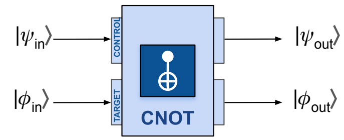


Inverting CNOT and CZ

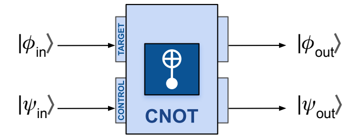
Quantum Circuit Structure

In a circuit, order and orientation of gates matters!

- *What happens to the matrix when you flip the orientation of CNOT?*



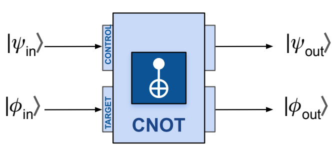
vs.



Review: The CNOT Gate

CNOT acts on two qubits: a control and a target. The input state is expressed as:

$$|\text{control}\rangle \otimes |\text{target}\rangle = |\psi_{in}\rangle \otimes |\phi_{in}\rangle = |\psi_{in}\phi_{in}\rangle$$

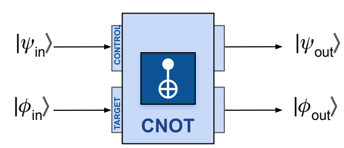


CNOT Symbol

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

CNOT Matrix

CNOT State Transformation



CNOT Symbol

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

CNOT Matrix

State Evolution:

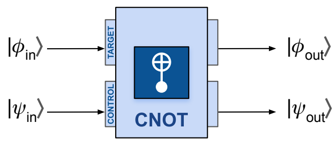
$$|\text{control}\rangle \otimes |\text{target}\rangle \mapsto |\text{control}\rangle \otimes |\text{target}\rangle$$
$$|\psi_{in}\rangle \otimes |\phi_{in}\rangle \mapsto |\psi_{out}\rangle \otimes |\phi_{out}\rangle$$

$$|0\rangle \otimes |0\rangle = |00\rangle \mapsto |0\rangle \otimes |0\rangle = |00\rangle$$
$$|0\rangle \otimes |1\rangle = |01\rangle \mapsto |0\rangle \otimes |1\rangle = |01\rangle$$
$$|1\rangle \otimes |0\rangle = |10\rangle \mapsto |1\rangle \otimes |1\rangle = |11\rangle$$
$$|1\rangle \otimes |1\rangle = |11\rangle \mapsto |1\rangle \otimes |0\rangle = |10\rangle$$

Invert the CNOT Gate

Quantum operations may be inverted in a circuit. What happens to their matrix?

- *Let's analyze input and output relations for the inverted CNOT!*



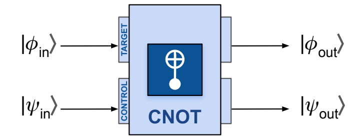
Inverted CNOT

State Evolution:

$|\text{target}\rangle \otimes |\text{control}\rangle \mapsto |\text{target}\rangle \otimes |\text{control}\rangle$
$|\phi_{in}\rangle \otimes |\psi_{in}\rangle \mapsto |\phi_{out}\rangle \otimes |\psi_{out}\rangle$

$|0\rangle \otimes |0\rangle = |00\rangle \mapsto |0\rangle \otimes |0\rangle = |00\rangle$
$|0\rangle \otimes |1\rangle = |01\rangle \mapsto |1\rangle \otimes |1\rangle = |11\rangle$
$|1\rangle \otimes |0\rangle = |10\rangle \mapsto |1\rangle \otimes |0\rangle = |10\rangle$
$|1\rangle \otimes |1\rangle = |11\rangle \mapsto |0\rangle \otimes |1\rangle = |01\rangle$

Invert the CNOT Gate



Inverted CNOT

$$\begin{bmatrix} ? \end{bmatrix}$$

Inverted CNOT Matrix

The inverted CNOT has a different matrix than the standard CNOT!

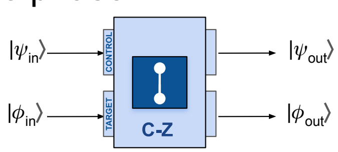
State Evolution:

$|\text{target}\rangle \otimes |\text{control}\rangle \mapsto |\text{target}\rangle \otimes |\text{control}\rangle$
$|\phi_{in}\rangle \otimes |\psi_{in}\rangle \mapsto |\phi_{out}\rangle \otimes |\psi_{out}\rangle$

$|0\rangle \otimes |0\rangle = |00\rangle \mapsto |0\rangle \otimes |0\rangle = |00\rangle$
$|0\rangle \otimes |1\rangle = |01\rangle \mapsto |1\rangle \otimes |1\rangle = |11\rangle$
$|1\rangle \otimes |0\rangle = |10\rangle \mapsto |1\rangle \otimes |0\rangle = |10\rangle$
$|1\rangle \otimes |1\rangle = |11\rangle \mapsto |0\rangle \otimes |1\rangle = |01\rangle$

CZ State Transformation

CZ acts on two qubits, a control and a target, selectively adding relative phase



CZ Symbol

State Evolution:

$|\text{control}\rangle \otimes |\text{target}\rangle \mapsto |\text{control}\rangle \otimes |\text{target}\rangle$
$|\psi_{in}\rangle \otimes |\phi_{in}\rangle \mapsto |\psi_{out}\rangle \otimes |\phi_{out}\rangle$

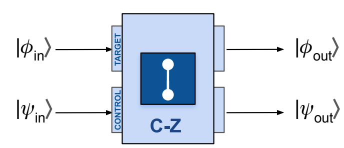
$|0\rangle \otimes |0\rangle = |00\rangle \mapsto |0\rangle \otimes |0\rangle = |00\rangle$
$|0\rangle \otimes |1\rangle = |01\rangle \mapsto |0\rangle \otimes |1\rangle = |01\rangle$
$|1\rangle \otimes |0\rangle = |10\rangle \mapsto |1\rangle \otimes |0\rangle = |10\rangle$
$|1\rangle \otimes |1\rangle = |11\rangle \mapsto |1\rangle \otimes -|1\rangle = -|11\rangle$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

CZ Matrix

Invert CZ

What happens when CZ is inverted?



State Evolution:

$|\text{target}\rangle \otimes |\text{control}\rangle \mapsto |\text{target}\rangle \otimes |\text{control}\rangle$
$|\phi_{in}\rangle \otimes |\psi_{in}\rangle \mapsto |\phi_{out}\rangle \otimes |\psi_{out}\rangle$

$|0\rangle \otimes |0\rangle = |00\rangle \mapsto |0\rangle \otimes |0\rangle = |00\rangle$
$|0\rangle \otimes |1\rangle = |01\rangle \mapsto |0\rangle \otimes |1\rangle = |01\rangle$
$|1\rangle \otimes |0\rangle = |10\rangle \mapsto |1\rangle \otimes |0\rangle = |10\rangle$
$|1\rangle \otimes |1\rangle = |11\rangle \mapsto -|1\rangle \otimes |1\rangle = -|11\rangle$

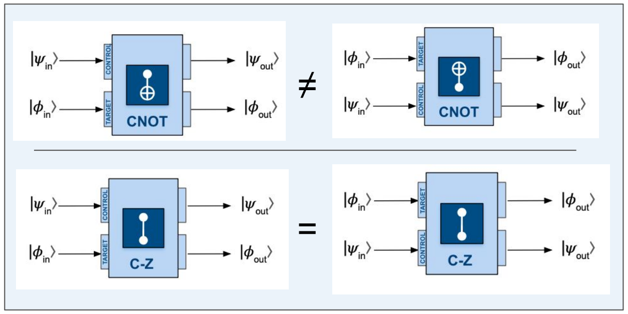
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

CZ Matrix

Inverted CZ has the same matrix!

Summary

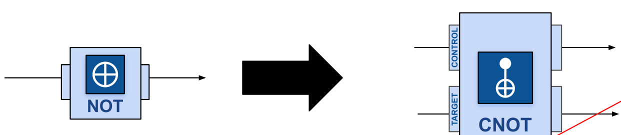
- Just like order matters for combining gates and qubits, orientation is important for multi-qubit operations
  - CNOT gate
- The CZ gate is special because inverting control and target results in the same qubit transformation and matrix



Controlled Gates and the CCNOT/Toffoli

Controlled Gates

Think about how the NOT gate becomes the CNOT gate



Examine the CNOT matrix. It contains:

1) An Identity Gate

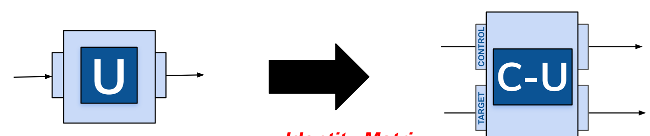
2) An NOT Gate

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Controlled Gates

- Creating a controlled gate from an arbitrary quantum gate U is possible.
- Add a second control qubit, and only apply gate if control has a probability amplitude for $|1\rangle$ (remember: control can be in superposition!):



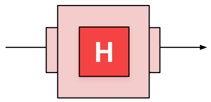
$$\begin{bmatrix} u_0 & u_1 \\ u_2 & u_3 \end{bmatrix}$$

Identity Matrix

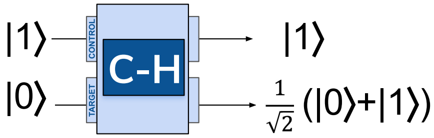
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & u_0 & u_1 \\ 0 & 0 & u_2 & u_3 \end{bmatrix}$$

U Matrix

Example: Controlled Hadamard



$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$



$$\textit{C-H} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{bmatrix}$$

State Calculation:

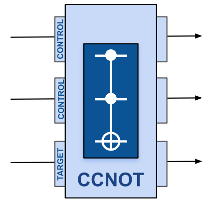
$$(\text{C-H})|10\rangle = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{bmatrix}\begin{bmatrix}0\\0\\1\\0\end{bmatrix} =$$

$$|1\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\end{bmatrix} \otimes \begin{bmatrix}1\\1\end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix}0\\0\\1\\1\end{bmatrix}$$

Additional Controls on CNOT

- The Controlled-CNOT, CCNOT, is also called the ***Toffoli*** gate
- The Toffoli acts on three qubits: two controls and one target
    - NOT operation on target if and only if both controls have a probability amplitude for |1⟩

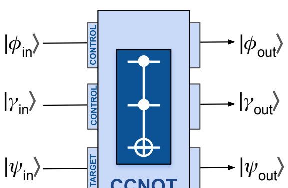


Two-qubit Identity Matrix, I⊗I

$$\text{CCNOT} = \begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\end{bmatrix}$$

CNOT Matrix

CCNOT State Transformation



$$\text{CCNOT} = \begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\end{bmatrix}$$

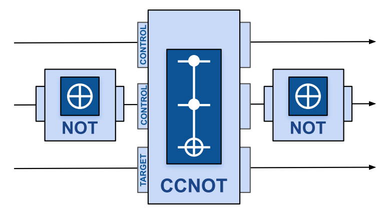
State Evolution:

$$|\phi_{in}\rangle \otimes |\gamma_{in}\rangle \otimes |\psi_{in}\rangle \mapsto |\phi_{out}\rangle \otimes |\gamma_{out}\rangle \otimes |\psi_{out}\rangle$$

|000⟩ ↦ |000⟩
|001⟩ ↦ |001⟩
|010⟩ ↦ |010⟩
|011⟩ ↦ |011⟩
|100⟩ ↦ |100⟩
|101⟩ ↦ |101⟩
|110⟩ ↦ |111⟩
|111⟩ ↦ |110⟩

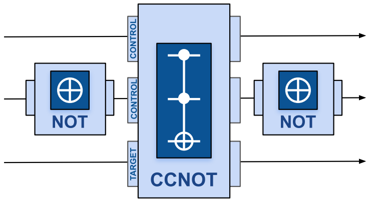
Note: The CCNOT gate with more control qubits is called a Generalized Toffoli

PRACTICE: What is the output corresponding to the input |100⟩ for the pictured quantum circuit ?



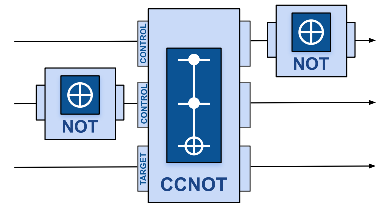
a. |101⟩
b. |100⟩
c. |110⟩
d. |111⟩

PRACTICE: What is the output corresponding to the input $|100\rangle$ for the pictured quantum circuit ?



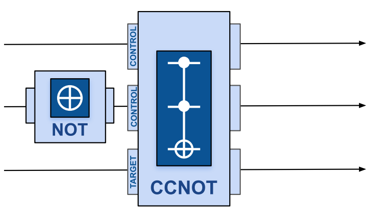
a. $|101\rangle$
b. $|100\rangle$
c. $|110\rangle$
d. $|111\rangle$

1) Consider the quantum circuit pictured here. If the input is $|110\rangle$, what is the output?



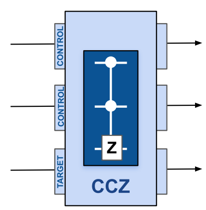
a. $|001\rangle$
b. $|000\rangle$
c. $|100\rangle$
d. $|111\rangle$

2) Consider the quantum circuit pictured here. If the input is $|110\rangle$, what is the output?



a. $|100\rangle$
b. $|101\rangle$
c. $|110\rangle$
d. $|111\rangle$

3) Which matrix represents a controlled-CZ operation?



a. $$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

b. $$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

c. $$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

d. $$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$