Quantum Teleportation

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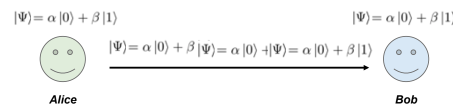
Protocol for transferring qubit from one party to another

Perfectly preserves state information



Quantum Teleportation

Allows for transmission over great distances...think from Earth to space!

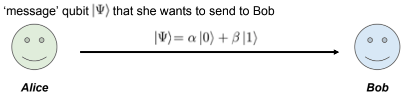
Not an exaggeration: [In 2017, qubits were teleported from earth to space!]

Does not involve:

- Moving matter from one point to another via dematerialization/ materialization
- Travelling forward or backward in time
- Travelling faster than the speed of light

Requirements for Quantum Teleportation

Alice has a 'message' qubit $|\Psi\rangle$ that she wants to send to Bob



Teleportation protocol requires additional resources:

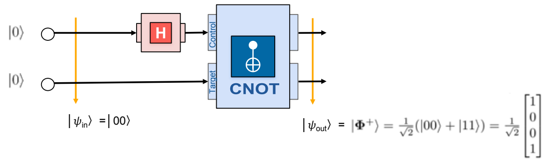
- Two entangled qubits...each party has one half of the entangled pair
- A classical communication line used to send two classical bits from Alice to Bob

Note: Since quantum teleportation needs classical communication, it cannot be faster than the speed of light!

Step 1: Create an entangled pair of qubits

Entangle two qubits using ***same entangle without phase***

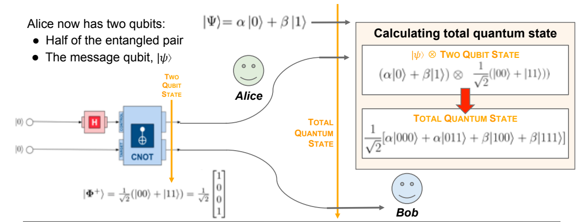
- Alice and Bob can entangle the qubits, or they can receive them from a third party!



Step 2: Distribute entangled qubits to Alice & Bob

Alice now has two qubits:

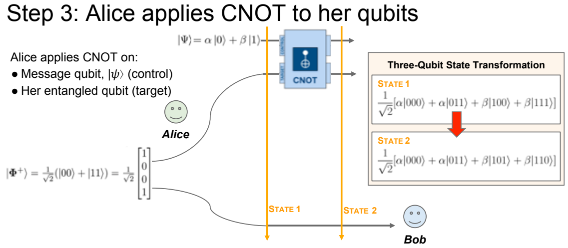
- Half of the entangled pair
- The message qubit, $|\psi\rangle$



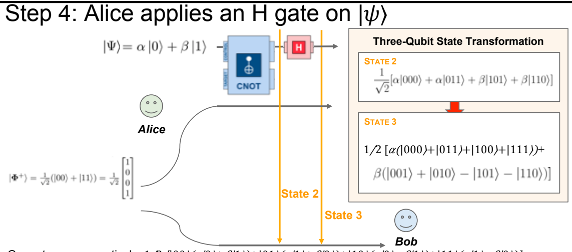
Step 3: Alice applies CNOT to her qubits

Alice applies CNOT on:

- Message qubit, $|\psi\rangle$ (control)
- Her entangled qubit (target)

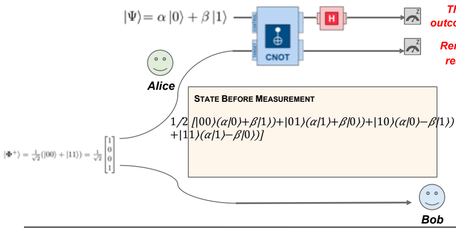


Step 4: Alice applies an H gate on $|\psi\rangle$



Group terms suggestively: $1/2\ [|00\rangle(\alpha|0\rangle+\beta|1\rangle)+|01\rangle(\alpha|1\rangle+\beta|0\rangle)+|10\rangle(\alpha|0\rangle-\beta|1\rangle)+|11\rangle(\alpha|1\rangle-\beta|0\rangle)]$
(Recall that Alice has first 2 qubits, Bob has last qubit)

Step 5: Alice measures both of her qubits



There are four possible outcomes for Alice's qubits…

Remember: measurement results in classical bits!

State Before Measurement

$1/2\,[|00\rangle(\alpha|0\rangle+\beta|1\rangle)+|01\rangle(\alpha|1\rangle+\beta|0\rangle)+|10\rangle(\alpha|0\rangle-\beta|1\rangle)+|11\rangle(\alpha|1\rangle-\beta|0\rangle)]$

Step 6: Process results of measurement

We deduce information about Bob's state by using partial measurements

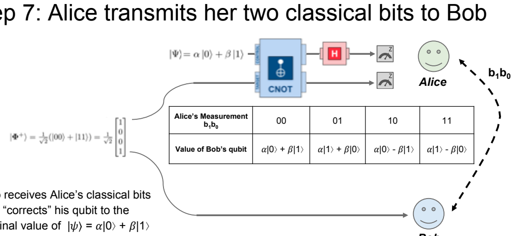
State Before Measurement

$1/2\,[|00\rangle(\alpha|0\rangle+\beta|1\rangle)+|01\rangle(\alpha|1\rangle+\beta|0\rangle)+|10\rangle(\alpha|0\rangle-\beta|1\rangle)+|11\rangle(\alpha|1\rangle-\beta|0\rangle)]$

Alice's Measurement	00	01	10	11
Value of Bob's qubit	$\alpha|0\rangle + \beta|1\rangle$	$\alpha|1\rangle + \beta|0\rangle$	$\alpha|0\rangle - \beta|1\rangle$	$\alpha|1\rangle - \beta|0\rangle$

Remember: Goal is to transmit $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$
If measurement result is 00, already done
For each other result 01, 10, or 11, need to apply particular correction

Step 7: Alice transmits her two classical bits to Bob



Alice's Measurement b_1b_0	00	01	10	11
Value of Bob's qubit	$\alpha|0\rangle + \beta|1\rangle$	$\alpha|1\rangle + \beta|0\rangle$	$\alpha|0\rangle - \beta|1\rangle$	$\alpha|1\rangle - \beta|0\rangle$

Bob receives Alice's classical bits and "corrects" his qubit to the original value of $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

How does Bob correct the state of $|\psi\rangle$?

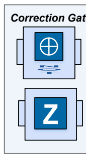
Step 8: Bob recovers $|\psi\rangle=\alpha|0\rangle + \beta|1\rangle$

To recover $|\psi\rangle$:

- if b_1 is 1, then apply a Z gate
- if b_0 is 1, then apply a NOT (X) gate

Alice's Measurement b_1b_0	00	01	10	11
Value of Bob's qubit	$\alpha|0\rangle + \beta|1\rangle$	$\alpha|1\rangle + \beta|0\rangle$	$\alpha|0\rangle - \beta|1\rangle$	$\alpha|1\rangle - \beta|0\rangle$
	No Gates Needed!	*Apply NOT Gate*	*Apply Z Gate*	*Apply NOT and Z Gate*

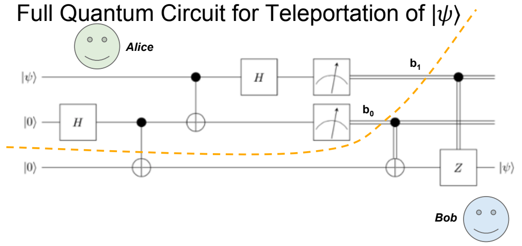
Correction Gates:



$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$

$\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$

Full Quantum Circuit for Teleportation of $|\psi\rangle$



PRACTICE:

Alice's Measurement b_1b_0	00	01	10	11
Value of Bob's qubit	$\alpha\|0\rangle + \beta\|1\rangle$	$\alpha\|1\rangle + \beta\|0\rangle$	$\alpha\|0\rangle - \beta\|1\rangle$	$\alpha\|1\rangle - \beta\|0\rangle$

During teleportation, Alice measures her qubits and sends Bob the bit string $\mathbf{b_1b_0}$ = 10. If the message qubit from Alice was intended to have a value of $|\psi\rangle$= 0.8|0⟩ + 0.6|1⟩, what is the value of the qubit currently in Bob's possession (before correction)?

a. 0.8|0⟩ + 0.6|1⟩
b. 0.8|0⟩ - 0.6|1⟩
c. 0.8|1⟩ + 0.6|0⟩
d. 0.8|1⟩ - 0.6|0⟩

PRACTICE:

Alice's Measurement b_1b_0	00	01	10	11
Value of Bob's qubit	$\alpha\|0\rangle + \beta\|1\rangle$	$\alpha\|1\rangle + \beta\|0\rangle$	$\alpha\|0\rangle - \beta\|1\rangle$	$\alpha\|1\rangle - \beta\|0\rangle$

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a. 0.8|0⟩ + 0.6|1⟩
b. 0.8|0⟩ - 0.6|1⟩
c. 0.8|1⟩ + 0.6|0⟩
d. 0.8|1⟩ - 0.6|0⟩

PRACTICE: How many entangled qubits (minimum) are needed to transmit a total of 5 message qubits with teleportation procedures?

a. 5 entangled qubits
b. 2 entangled qubits
c. 10 entangled qubits
d. 15 entangled qubits

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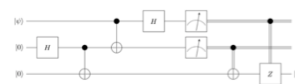
Notes about QuantumTeleportation



- Alice never knows the state of $|\psi\rangle$
- Communication is not faster than light because of the need for a classical communication channel
- Alice and Bob destroy their entangled qubits during the process of teleportation... more arbitrary qubit transfer requires more distributed entanglement!
- Quantum teleportation has been experimentally demonstrated many times
- Only need to transmit 2 classical bits (and start with shared entangled pair)

Allows transmitting arbitrary $|\psi\rangle$

Applications of Teleportation

Quantum teleportation can be used in many future quantum computing tasks!
Projected applications include:

- Reducing computation errors
  - Noise-resistant quantum gates
  - Error correcting codes
- Uniting quantum computers to form networks
- Constructing ultra-secure communication channels
  - Qubits are transferred with ultimate privacy...eavesdroppers cannot read messages

The No-Cloning Theorem of QIS

Copying Information

Classical computing relies on copying information for:

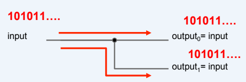
- Computation
- Data storage
- Error detection and correction

Examples of copying classical data include:

```python
my_var = 1
copy_my_var = my_var
print(f'Original: {my_var}, Copy:{copy_my_var}')
```

```
Original: 1, Copy:1
```

Assigning a new variable the value of an existing variable in Python

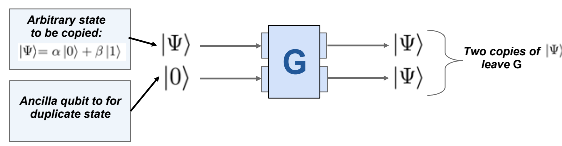


Sending one signal down multiple wires with voltage fan-out in a classical circuit

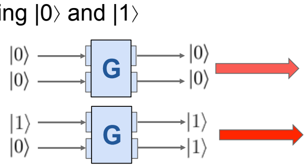
"Cloning" a qubit

Can qubit state be cloned or duplicated?

Let's attempt to define a "qubit copying" circuit, **G**:



Copying |0〉 and |1〉



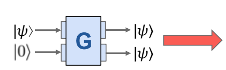
G$|00\rangle = |00\rangle$

G$|10\rangle = |11\rangle$

Seems that a function can be defined for copying qubit basis states (computational basis).

*Can **superposition** be copied?*

Copying Arbitrary Quantum State, $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$



G$|\psi 0\rangle \stackrel{?}{=} |\psi\psi\rangle$

Left side of equation: **G**$|\psi 0\rangle$ where $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

Expand: **G**$|\psi 0\rangle$ = **G**$(|\psi\rangle \otimes |0\rangle)$ = **G**$((\alpha|0\rangle + \beta|1\rangle) \otimes |0\rangle)$

Distribute: **G**$(\alpha|0\rangle \otimes |0\rangle + \beta|1\rangle \otimes |0\rangle)$

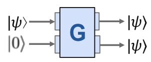
Simplify: **G**$(\alpha|00\rangle + \beta|10\rangle)$ = **G**$(\alpha|00\rangle)$ + **G**$(\beta|10\rangle)$

Apply Copy Gate, G: **G**$(\alpha|00\rangle)$ + **G**$(\beta|10\rangle)$ = $\alpha|00\rangle + \beta|11\rangle$

Resulting Quantum State after Copy Gate, G

Qubit state of $|\psi\rangle$ copied to ancilla qubit $|0\rangle$

Copying Arbitrary Quantum State, $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$



$G|\psi 0\rangle \stackrel{?}{=} |\psi\psi\rangle$

Right side of equation: $|\psi\psi\rangle$ where $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

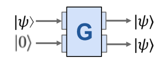
Expand: $|\psi\psi\rangle = |\psi\rangle \otimes |\psi\rangle = (\alpha|0\rangle + \beta|1\rangle)\otimes(\alpha|0\rangle + \beta|1\rangle)$

Distribute (FOIL): $(\alpha|0\rangle + \beta|1\rangle)\otimes(\alpha|0\rangle + \beta|1\rangle)$

$= \alpha|0\rangle\alpha|0\rangle + \alpha|0\rangle\beta|1\rangle + \beta|1\rangle\alpha|0\rangle + \beta|1\rangle\beta|1\rangle$

Simplify: $\alpha^2|00\rangle + \alpha\beta|01\rangle + \alpha\beta|10\rangle + \beta^2|11\rangle$ ← ***Calculated value of*** $|\psi\psi\rangle$

Copying Arbitrary Quantum State, $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$



$G|\psi 0\rangle \neq |\psi\psi\rangle$

***Anticipated output after state copy*:** $\alpha^2|00\rangle + \alpha\beta|01\rangle + \alpha\beta|10\rangle + \beta^2|11\rangle$

***Output produced by copy gate, G*:** $\alpha|00\rangle + \beta|11\rangle$

There is no copy (clone) gate that can duplicate qubit state!

Takeaway: The No-Cloning Theorem

- Qubits cannot be duplicated...we call this the No-cloning Theorem
  - Cannot 'see' quantum state without destroying it! Similar to measurement...
- The No-cloning Theorem has major implications on the use and storage of quantum information
- Major differences will exist for quantum versions of:
  - Algorithms
  - Error correction and detection
  - Memory

We must rethink how to solve problems as compared to classical approaches if we want to use the unique properties of QIS such as superposition!

This material is based upon work supported by the National Science Foundation under Grants No. 1730088 and No. 1730449. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.