The third dimension

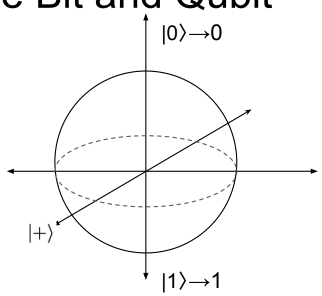
Visualizing the Bit and Qubit

Classical Bit - only possible states are 0 and 1 represented as endpoints of a line (no in between!)

Partial Qubit State- only possible states are 0 and 1 represented as endpoints of a line or a superposition of the two

(no phase)

Visualizing the Bit and Qubit



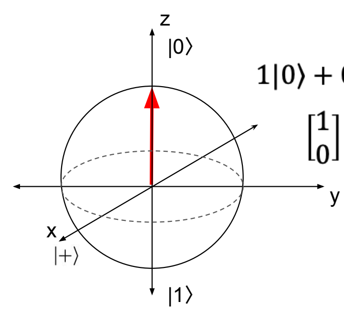
Full Unentangled Qubit State -

z axis is probability of measuring 0 vs 1,

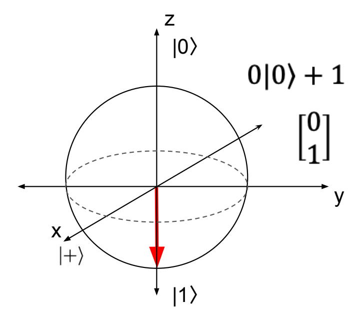
x axis is positive, negative phase

y axis is i, negative i phase

Qubit States on the Bloch Sphere

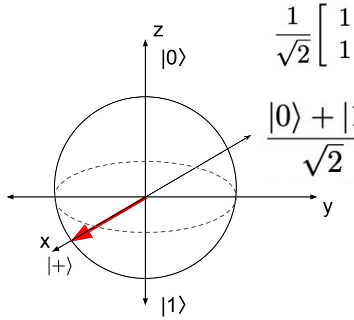


$1|0\rangle + 0|1\rangle$ = $\begin{bmatrix}1\\0\end{bmatrix}$

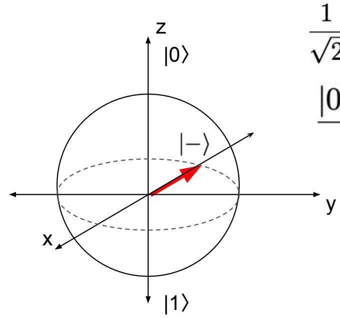


$0|0\rangle + 1|1\rangle$ = $\begin{bmatrix}0\\1\end{bmatrix}$

Qubit States on the Bloch Sphere



$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

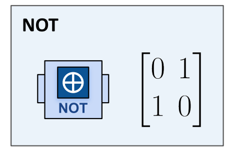


$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix} = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

Note: Resting on the equator of the Bloch Sphere results in a 50/50 probability of measuring either |0⟩ or |1⟩

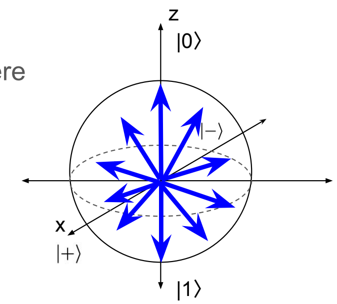
Single-Qubit Operations

- Qubit state represented with the Bloch Sphere
- Qubit gates we have learned about include:



|1⟩ = NOT |0⟩

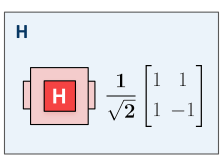
|0⟩ = NOT |1⟩



The NOT gate is also called the X gate...
It implements a rotation of π around the x-axis of the Bloch Sphere!

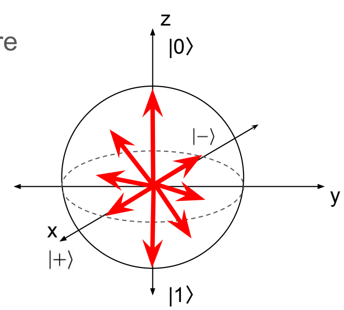
Single-Qubit Operations

- Qubit state represented with the Bloch Sphere
- Qubit gates we have learned about include:



|+⟩= 1/√2(|0⟩+|1⟩) = H |0⟩

|-⟩= 1/√2(|0⟩-|1⟩) = H |1⟩



The H gate implements a rotation of π/2 around the y-axis of the Bloch Sphere!

Why is this useful?

Perform arbitrary operations

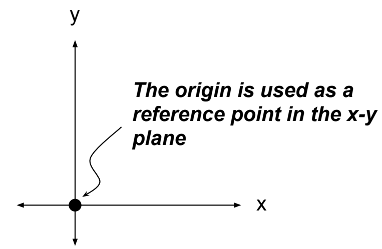
All operators are a rotation in some way

Some quantum computers can implement some gates globally and some locally, so a single (locally unsupported) gate needs to be changed into global gate, local gate, opposite global gate

Coordinate Systems:
2D to 3D

Coordinate Systems

- Must determine an object's 'address' in a space relative to a known position, the ***origin***
- Particular applications make description with one coordinate system/dimension easier than another!
  - Are you working with a grid? Or curved lines, surfaces, or spaces?
- Trigonometric identities help us convert between coordinate systems



Trigonometry: Relating Lengths and Angles!

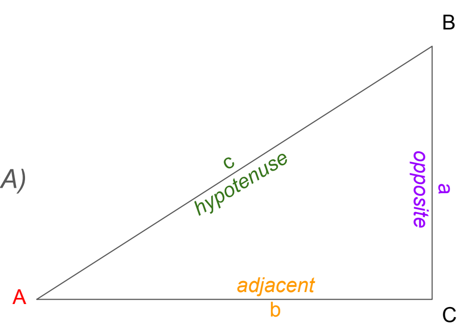
The trigonometric functions of angle A are:

- Sin(A) = opposite/hypotenuse = a/c
- Cos(A) = adjacent/hypotenuse = b/c
- Tan(A) = opposite/adjacent = a/b

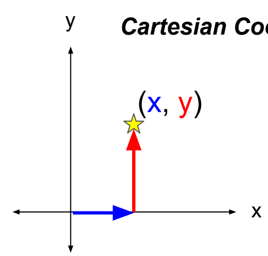
(hint: SOHCAHTOA)

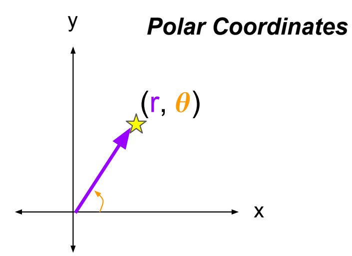
Pythagorean Theorem:

- $a^2 + b^2 = c^2$



Coordinate Systems in Two Dimensions (2D)





- Position of data point defined by distance in x and y directions from origin
- Reference axes are orthogonal to each other (right angles)

- Position of data point given as coordinates of r and θ
- Value r is the direct distance from the origin (the radius), and θ is angle from the x-axis

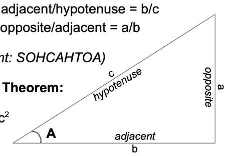
Cartesian and Polar Conversion

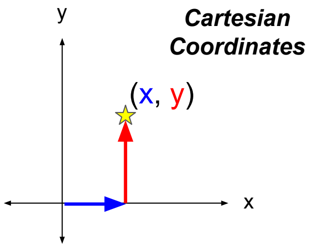
- Sin(A) = opposite/hypotenuse = a/c
- Cos(A) = adjacent/hypotenuse = b/c
- Tan(A) = opposite/adjacent = a/b

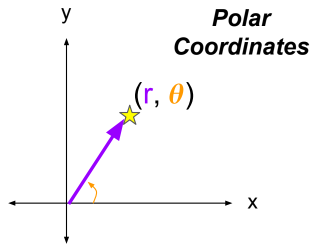
(hint: SOHCAHTOA)

Pythagorean Theorem:

- $a^2 + b^2 = c^2$







Cartesian to Polar: $r^2 = x^2 + y^2$ and $\tan(\theta) = y/x$

Polar to Cartesian: $x = r*\cos(\theta)$ and $y = r*\sin(\theta)$

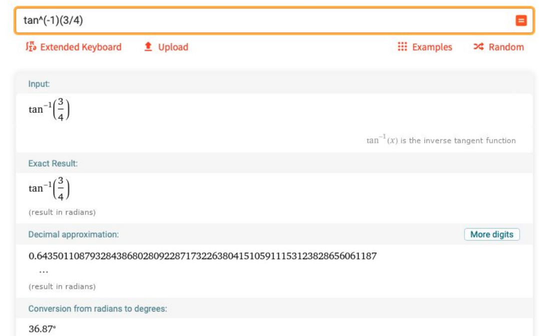
PRACTICE: Convert 2D Cartesian coordinates (x, y) = (4, 3) into Polar coordinates, (r, θ).

Cartesian to Polar: $r^2 = x^2 + y^2$ a

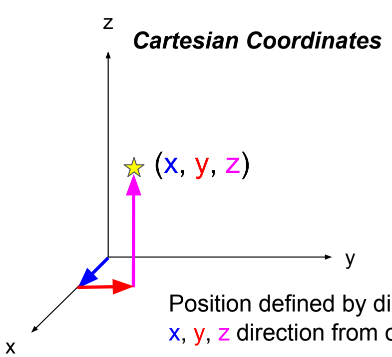
Polar to Cartesian: $x = r*\cos(\theta)$.

r^2 = 4^2 + 3^2
r^2 = 25
r = 5

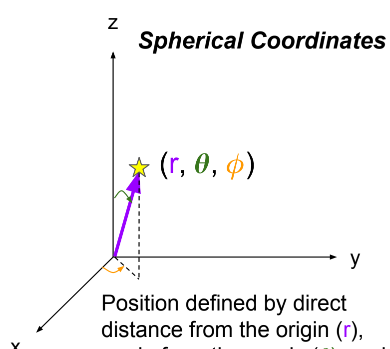
tan(θ) = 3/4
θ = 0.643



Coordinate Systems in Three Dimensions (3D)

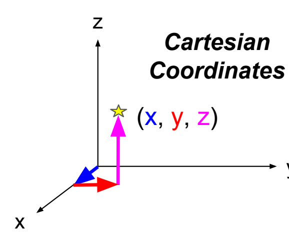


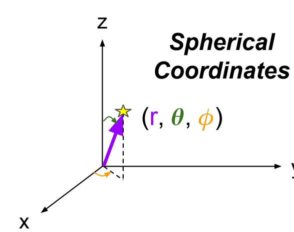
Position defined by distance in x, y, z direction from origin



Position defined by direct distance from the origin (r), angle from the z-axis (θ), and angle from x-axis (ϕ)

Cartesian to Spherical Coordinates



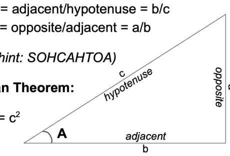


- Sin(A) = opposite/hypotenuse = a/c
- Cos(A) = adjacent/hypotenuse = b/c
- Tan(A) = opposite/adjacent = a/b

(hint: SOHCAHTOA)

Pythagorean Theorem:

- $a^2 + b^2 = c^2$



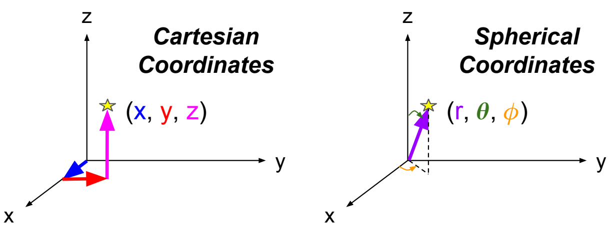
Cartesian to Spherical:

$r^2 = x^2 + y^2 + z^2$, $\tan(\phi) = y/x$, $\tan(\theta) = \mathrm{sqrt}(x^2 + y^2)/z$

Extension of Pythagorean Theorem

Opposite/Adjacent

Spherical to Cartesian Coordinates



- Sin(A) = opposite/hypotenuse = a/c
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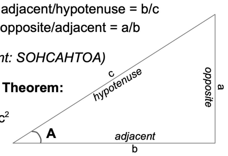
Pythagorean Theorem:

- $a^2 + b^2 = c^2$

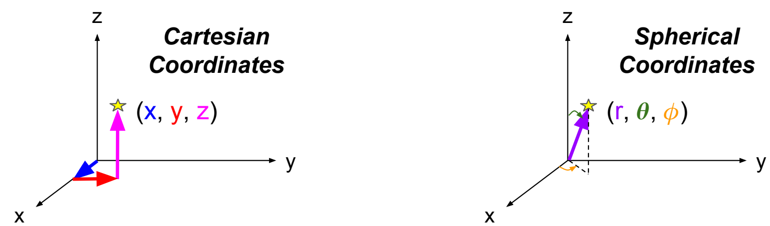


Spherical to Cartesian:

x = r*sin(θ)cos(ϕ) , y = r*sin(θ)sin(ϕ) , z = r*cos(θ)

*(sqrt($x^2 + y^2$)/r) * (x/sqrt($x^2 + y^2$))*   *(sqrt($x^2 + y^2$)/r) * (y/sqrt($x^2 + y^2$))*   *(z/r)*

Cartesian and Spherical Conversions



Cartesian to Spherical:

$r^2 = x^2 + y^2 + z^2$, tan(ϕ) = y/x , tan(θ) = sqrt($x^2 + y^2$)/z

Spherical to Cartesian:

x = r*sin(θ)cos(ϕ) , y = r*sin(θ)sin(ϕ) , z = r*cos(θ)

PRACTICE: Are the 3D Cartesian Coordinates (x, y, z) = (0,0,1) equal to the Spherical Coordinates (r, θ, ϕ) = (1,0,0)?

Cartesian to Spherical:

$r^2 = x^2 + y^2 + z^2$, tan(ϕ) = y/x , tan(θ) = sqrt($x^2 + y^2$)/z

Spherical to Cartesian:

x = r*sin(θ)cos(ϕ) , y = r*sin(θ)sin(ϕ) , z = r*cos(θ)

A. Yes
B. No, it's equal to (r, θ, ϕ) = (1, pi, pi/2)
C. No, it's equal to (r, θ, ϕ) = (1, pi, pi/3)
D. No, it's equal to (r, θ, ϕ) = (1, pi/2,pi/2)

PRACTICE: Are the 3D Cartesian Coordinates (0,0,1) equal to the Spherical Coordinates (1,0,0)?

Cartesian to Spherical:

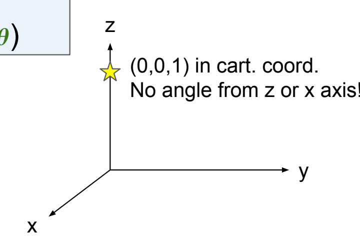
$r^2 = x^2 + y^2 + z^2$, tan(ϕ) = y/x , tan(θ) = sqrt($x^2 + y^2$)/z

Spherical to Cartesian:

x = r*sin(θ)cos(ϕ) , y = r*sin(θ)sin(ϕ) , z = r*cos(θ)

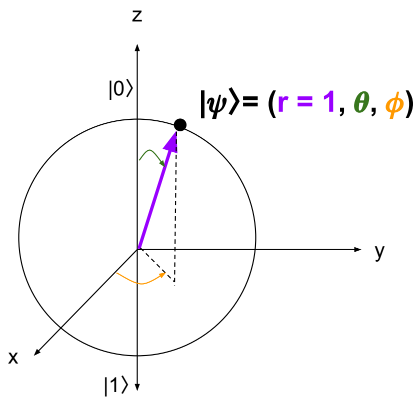
A. YES! tan^(-1)(0) = 0 rad, 0 degrees

Also, think of where this point is located in 3D space….



Relation to Quantum Computing

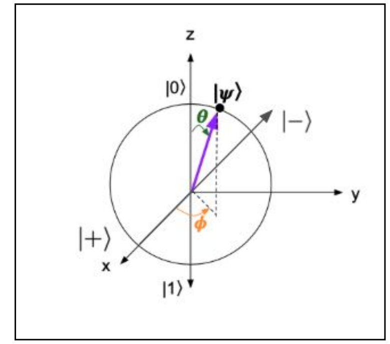
- Qubits have three attributes, two of which we have covered already
- The ***Bloch Sphere*** depicts the single qubit as a point on a sphere
  - All valid quantum states are on the surface of the sphere at radius 1 from origin
  - Some quantum operations are reflections or rotations around an axis
  - Other quantum operations are arbitrary rotations of θ and ϕ



PRACTICE: What are the angles θ and ϕ for the $|+\rangle$ state?

$$|+\rangle \quad \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

A. θ = pi/2 and ϕ = pi/2
B. θ = pi and ϕ = pi/2
C. θ = 0 and ϕ = pi/2
D. θ = pi/2 and ϕ = 0



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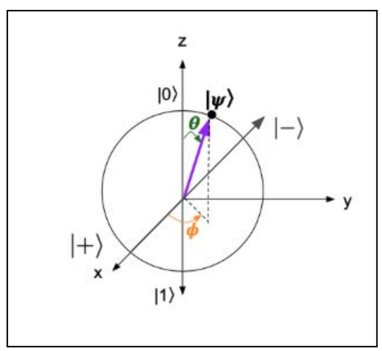
C. θ = 0 and ϕ = pi/2
D. θ = pi/2 and ϕ = 0



PRACTICE: The Bloch Sphere Radius is equal to 1 because….

A. Math is easier this way
B. Probabilities associated with basis states must sum to one (hundred percent chance you must be SOMEWHERE…)
C. It doesn't have to!
D. $|1\rangle$ is a basis state