Last Time:
- Finish Simon's algorithm
- Finish Shor's algorithm
- Start Grover's algorithm

Today:
- Finish Grover's algorithm
- Advanced mathematical concepts

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1

# Grover's Algorithm

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2

## Unstructured Search Problem

Suppose have database with $N$ entries
- where $N$ is very large
- Single entry is "marked"
  - Special entry we are looking for

Ex: database has one entry for each human currently alive
- approx. 8 billion
- entries are (name, birthdate)
- marked entry: person with name "Lov Grover"

For classical computer, may take $N$ database queries to find marked entry

For quantum computer, can do this in $\sqrt{N}$ queries using Grover's algorithm

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3

## Amplitude Amplification

Grover's algorithm implements ***amplitude amplification*** to increase the probability of observing the correct answer (the object of the search)

- **INCREASES** probability amplitude associated with answer $|\text{ket}\rangle$
- **DECREASES** all other probability amplitudes

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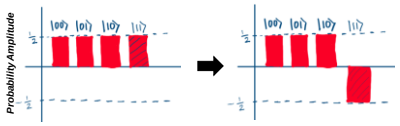
4

## Amplitude Amplification Procedure

**Step 1**: Start with a balanced superposition, and assign a phase of -1 to the chosen ket, |11⟩

BALANCED SUPERPOSITION
$|\psi\rangle = \frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle$



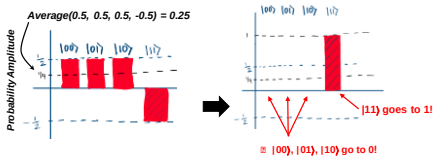

**We will call the matrix that implements the phase-flip $U_f$**



5

## Amplitude Amplification Procedure

**Step 2**: Invert all probability amplitudes about the mean (average)




**We will call the matrix that inverts around the mean $U_\varphi$**

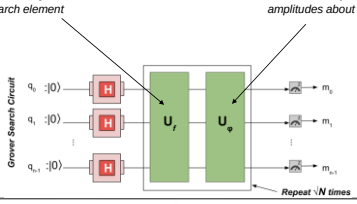


6

## Amplitude Amplification: Two Parts

*The **Oracle** flips the sign of the desired search element*

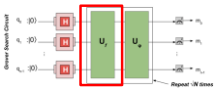
*The **Diffusion Operator** inverts amplitudes about the mean*






7

## Building the Grover Oracle: Bird's Eye View




1. Classical function

$$f(x) = \begin{cases} 0, \text{ if input x doesn't satisfy criteria } \times \\ 1, \text{ if x input satisfies criteria } \checkmark \end{cases}$$

2. Create reversible circuit that calculates f(x)
   Recall: can convert irreversible circuit to reversible circuit using ancilla

3. Add an additional ancilla that introduces *phase kickback* when f(x) = 1
   Object of search gets phase of -1



8

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