Hash Table (cont.)

CS143: lecture 12

Byron Zhong, July 11

Hash Table

Recap

- Nice $O(1)$ complexity because we can index into an array instead of chasing pointers
- We have a way to turn anything into an integer -- hash function
- We have a way to force any integers into a reasonable range -- compression (usually modulus)
- We need to handle collisions:
  - Collisions can be the result of the hash function
  - ... of compression

Hash Table

Handling Collision

- Two approaches:
  1. Chaining
  2. Probing

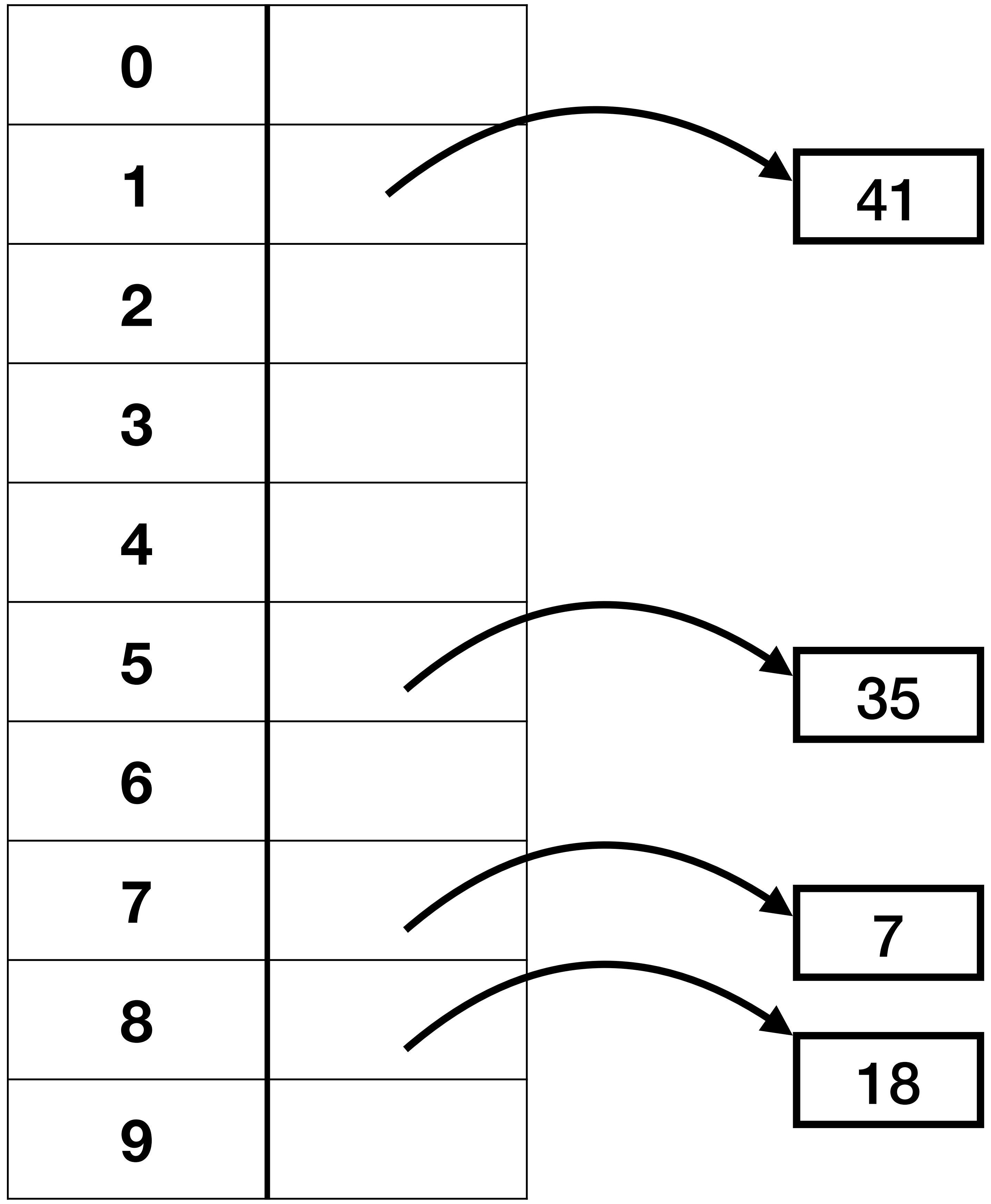
Chaining

- Each slot is a *list* of key-value pairs, called a *bucket*

0	
1	41
2	
3	
4	
5	35
6	
7	7
8	18
9	

Chaining

- Each slot is a *list* of key-value pairs, called a *bucket*

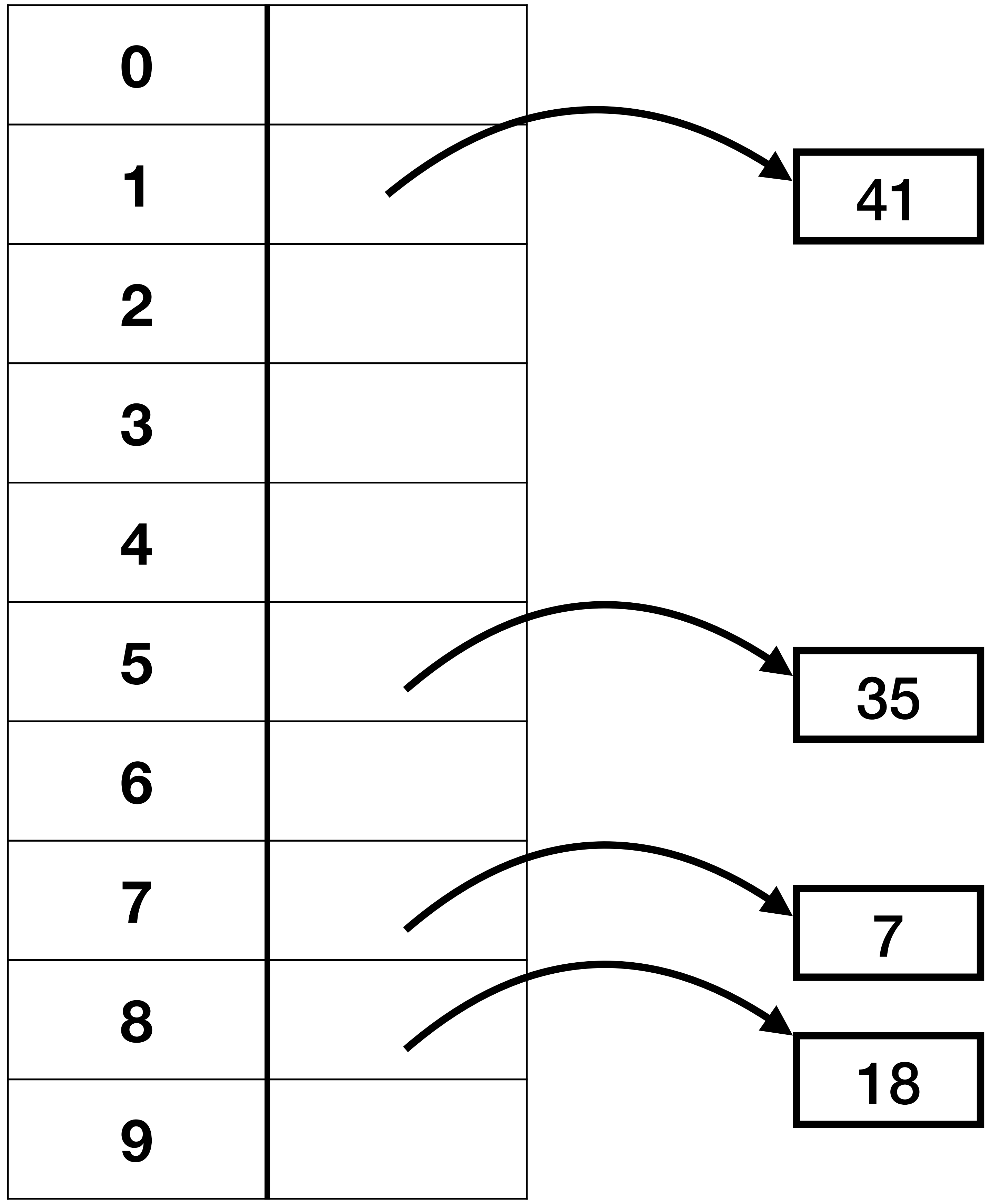


- You can use either list implementation
  - ...but there is an obvious choice
  - linked list, because of deletion

Chaining

Insert

- Each slot is a *list* of key-value pairs, called a *bucket*

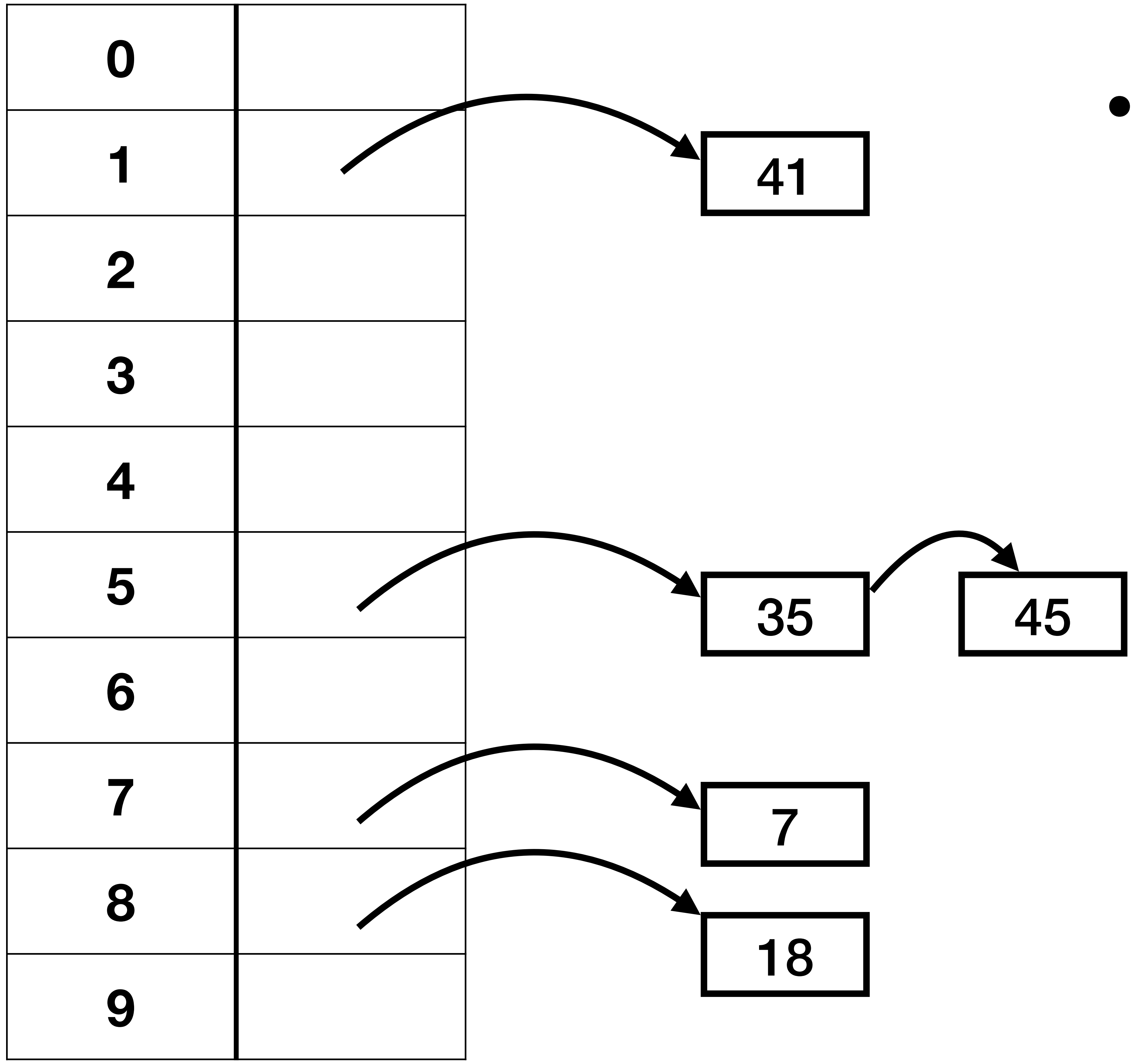


- Collisions will be prepended into the list

Chaining

Insert

- Each slot is a *list* of key-value pairs, called a *bucket*



- Collisions will be prepended into the list

Chaining

Insert

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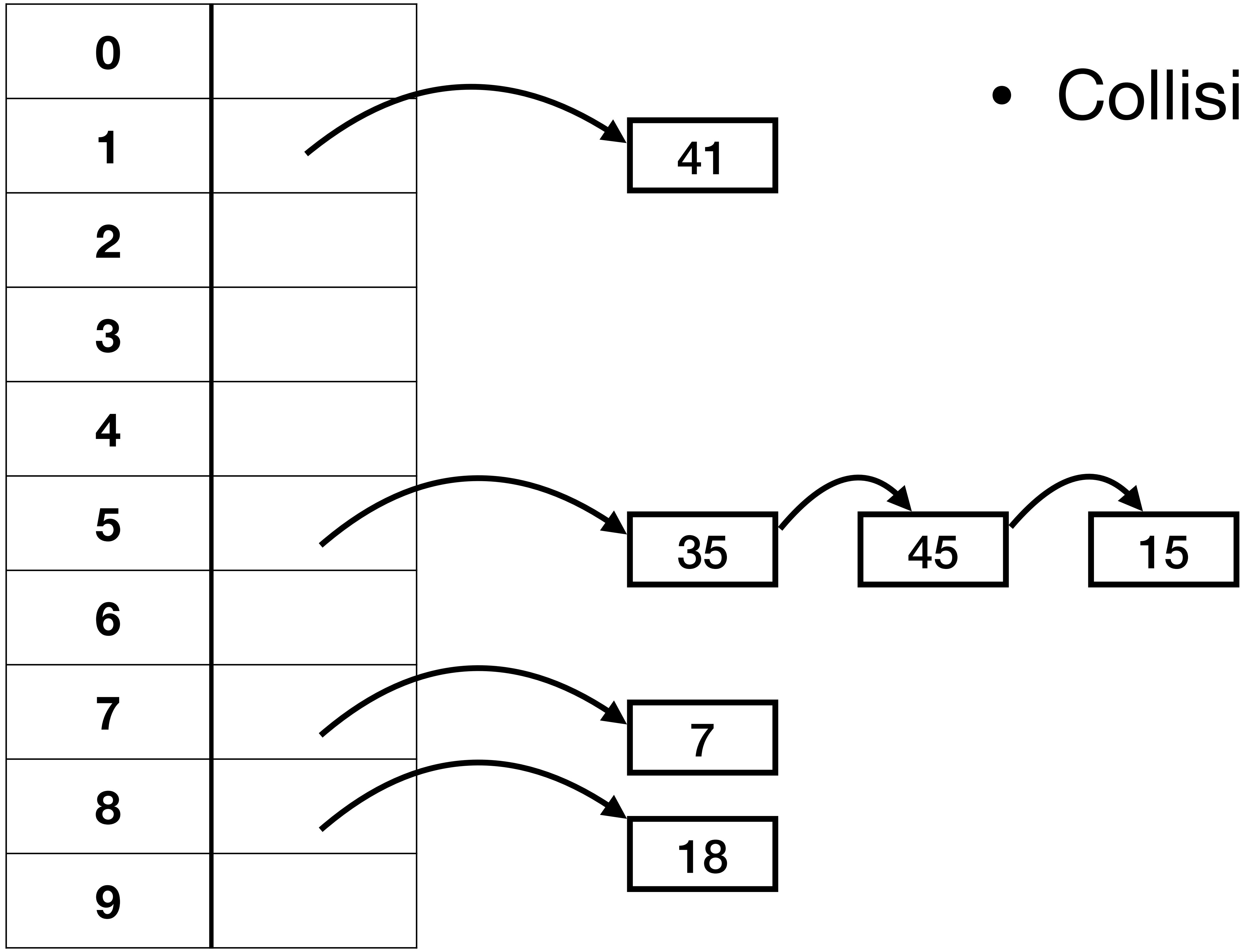
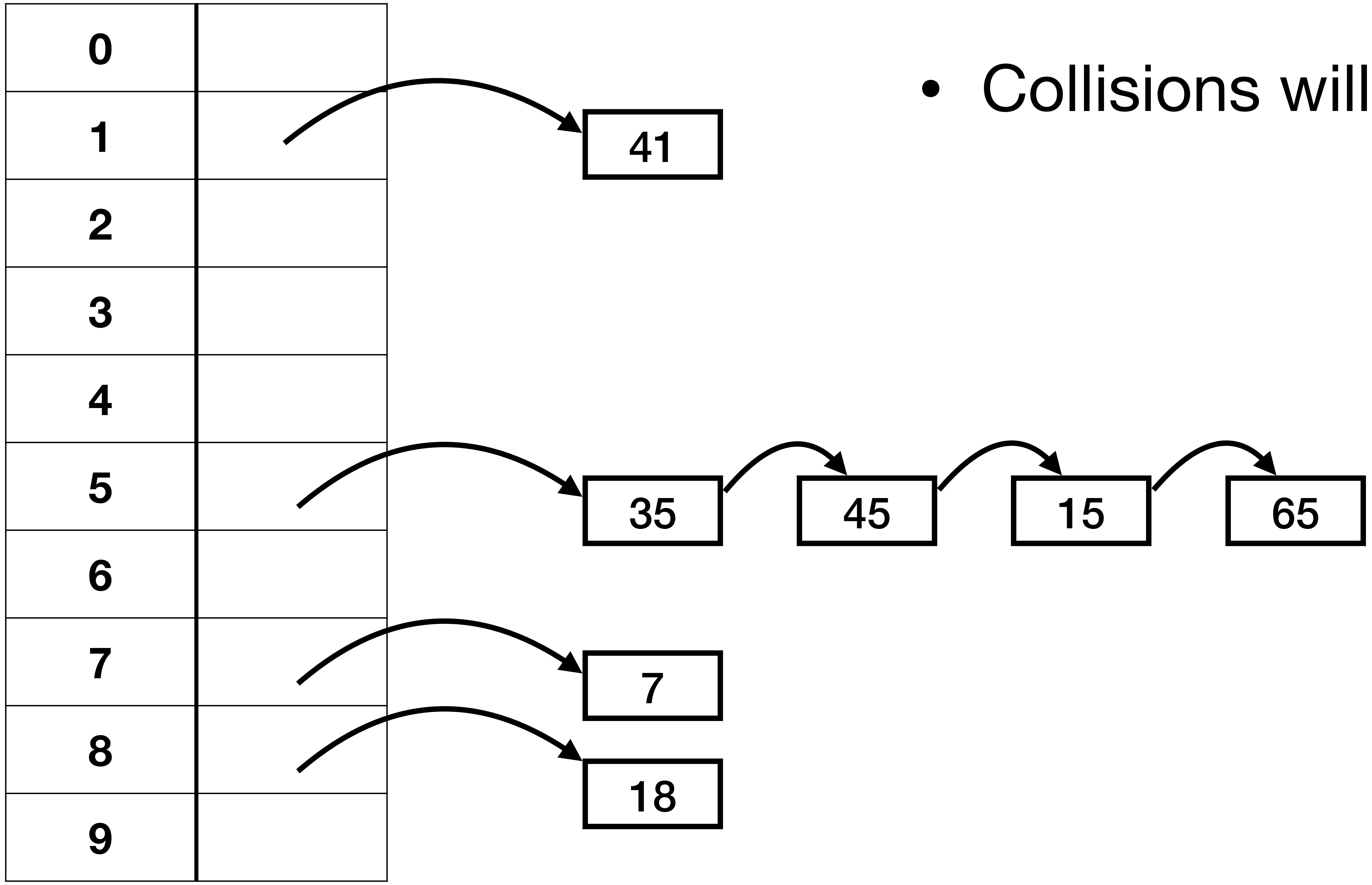


- Collisions will be prepended into the list

Chaining

Insert

- Each slot is a *list* of key-value pairs, called a *bucket*

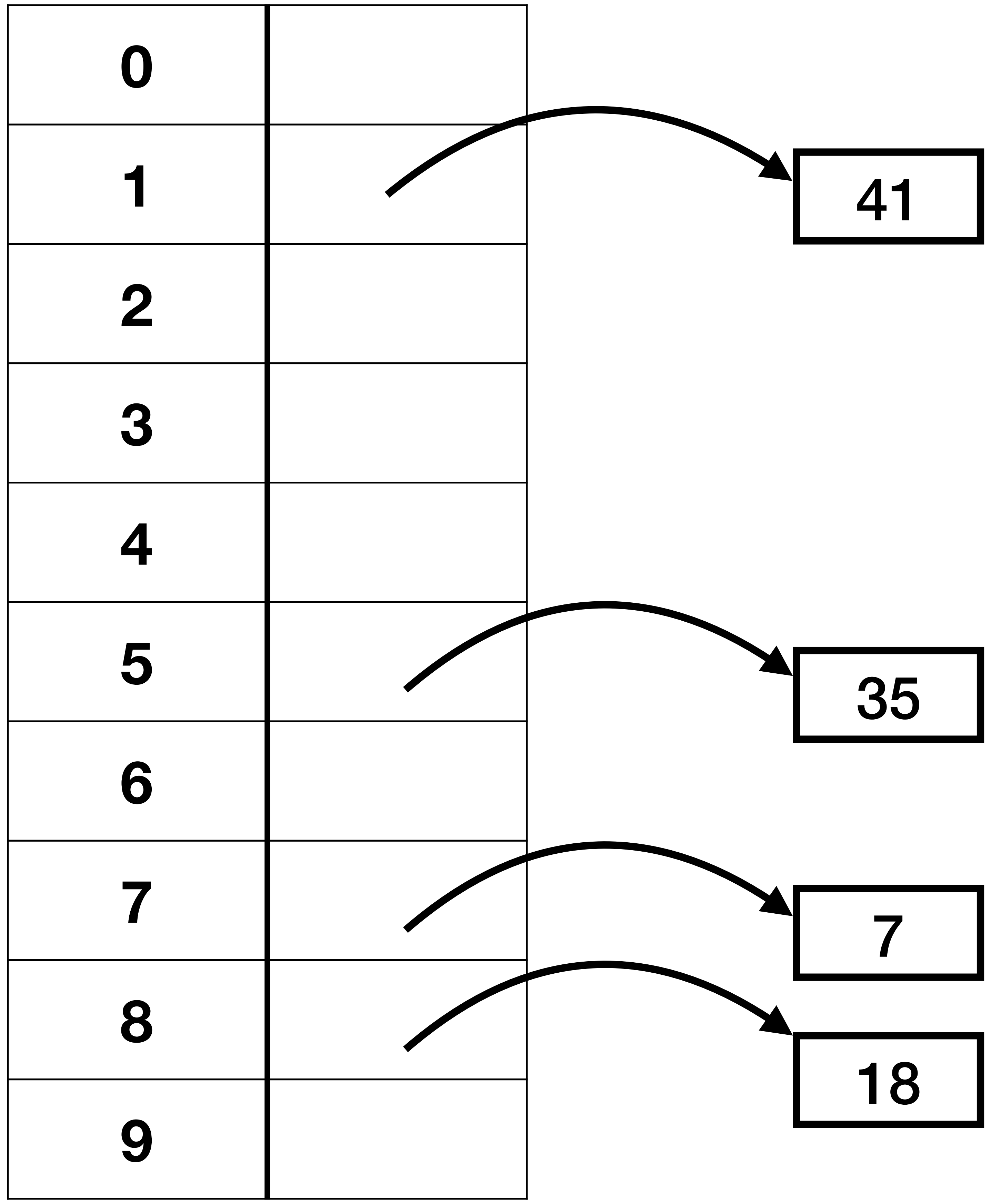


- Collisions will be prepended into the list

Chaining

Insert

- Each slot is a *list* of key-value pairs, called a *bucket*

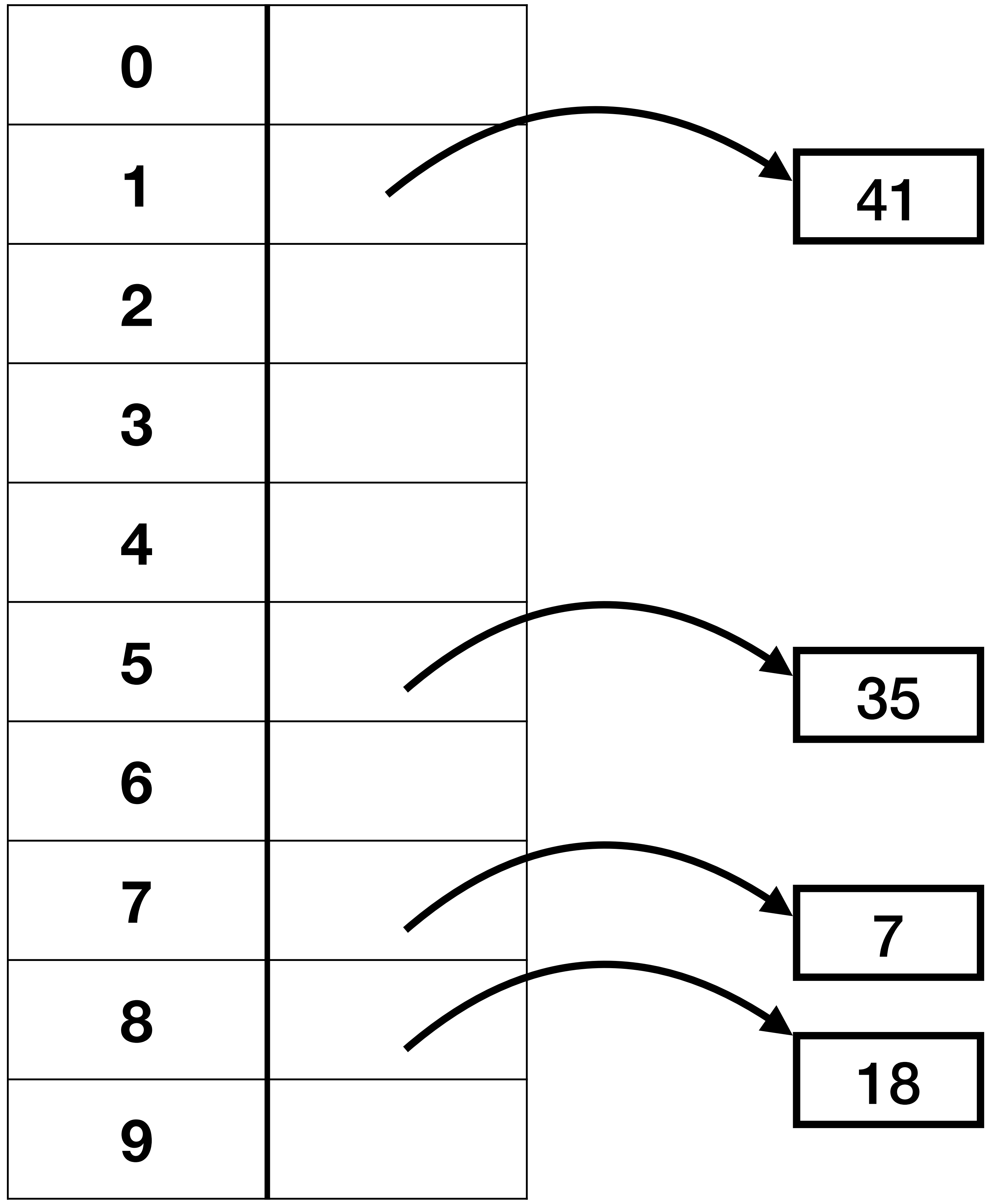


```
insert(table, key, value):
 bucket_idx = hash(key) % table->size
 if found key in table->buckets[bucket_idx]
  replace value
 else:
  add (key, value) into the list
```

Chaining

Insert

- Each slot is a *list* of key-value pairs, called a *bucket*



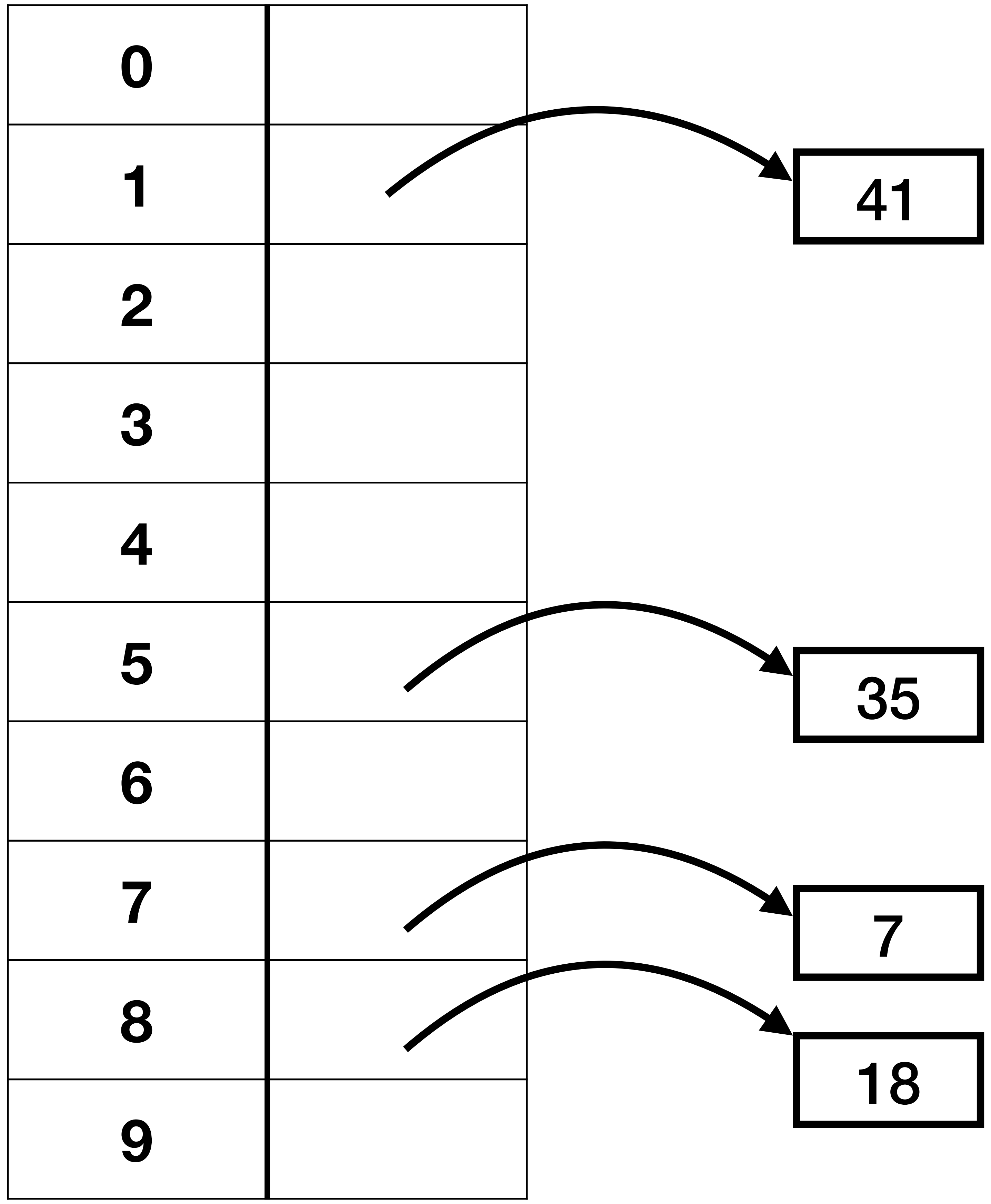
```
struct table {
        int size;
        int length;
        int (*eq)(void *, void *);
        uint64_t (*hash)(void *);
        struct bucket *buckets[];
};

struct bucket {
        void *key;
        void *value;
        struct bucket *next;
};
```

Chaining

Insert

- Each slot is a *list* of key-value pairs, called a *bucket*



```
struct table {
        int size;
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        int (*eq)(void *, void *);
        uint64_t (*hash)(void *);
        struct bucket *buckets[];
};

struct bucket {
        void *key;
        void *value;
        struct bucket *next;
};
```

<<-- what is this?

Interlude

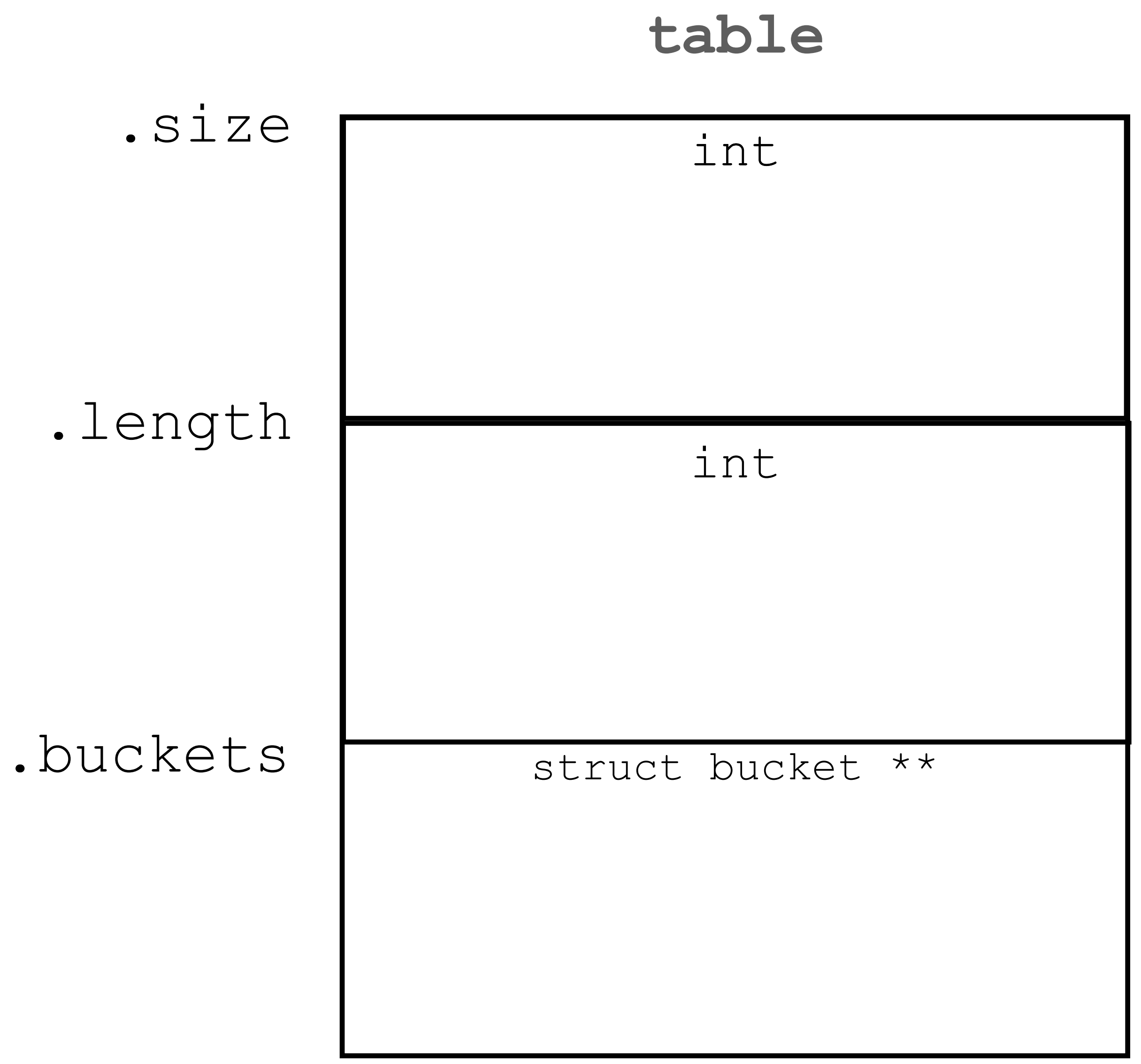
Flexible array member

- The last element of a structure may have an incomplete array type (empty bracket)
- `sizeof` does not include the incomplete field
- Why?

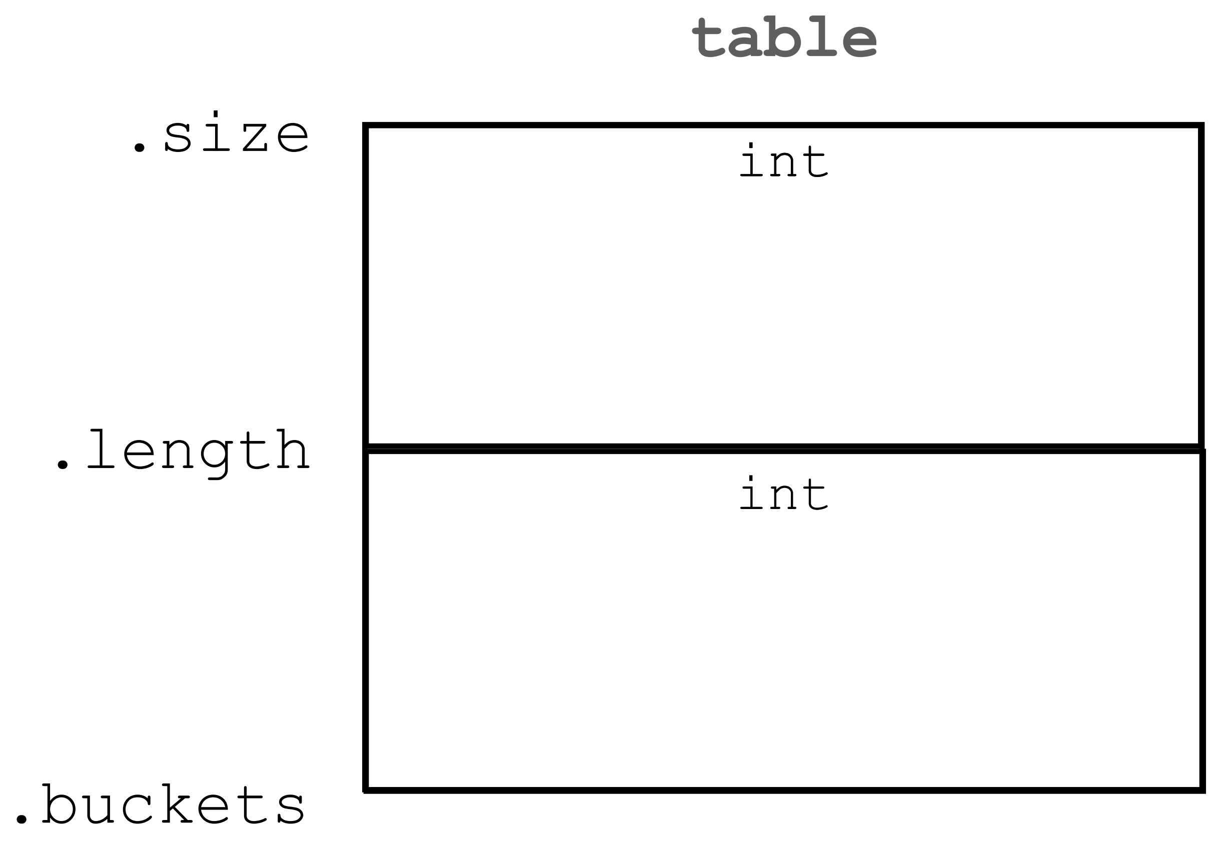
Interlude: Flexible Array Member

Memory Layout

```
struct table {
        int size;
        int length;
        struct bucket **buckets;
};
```



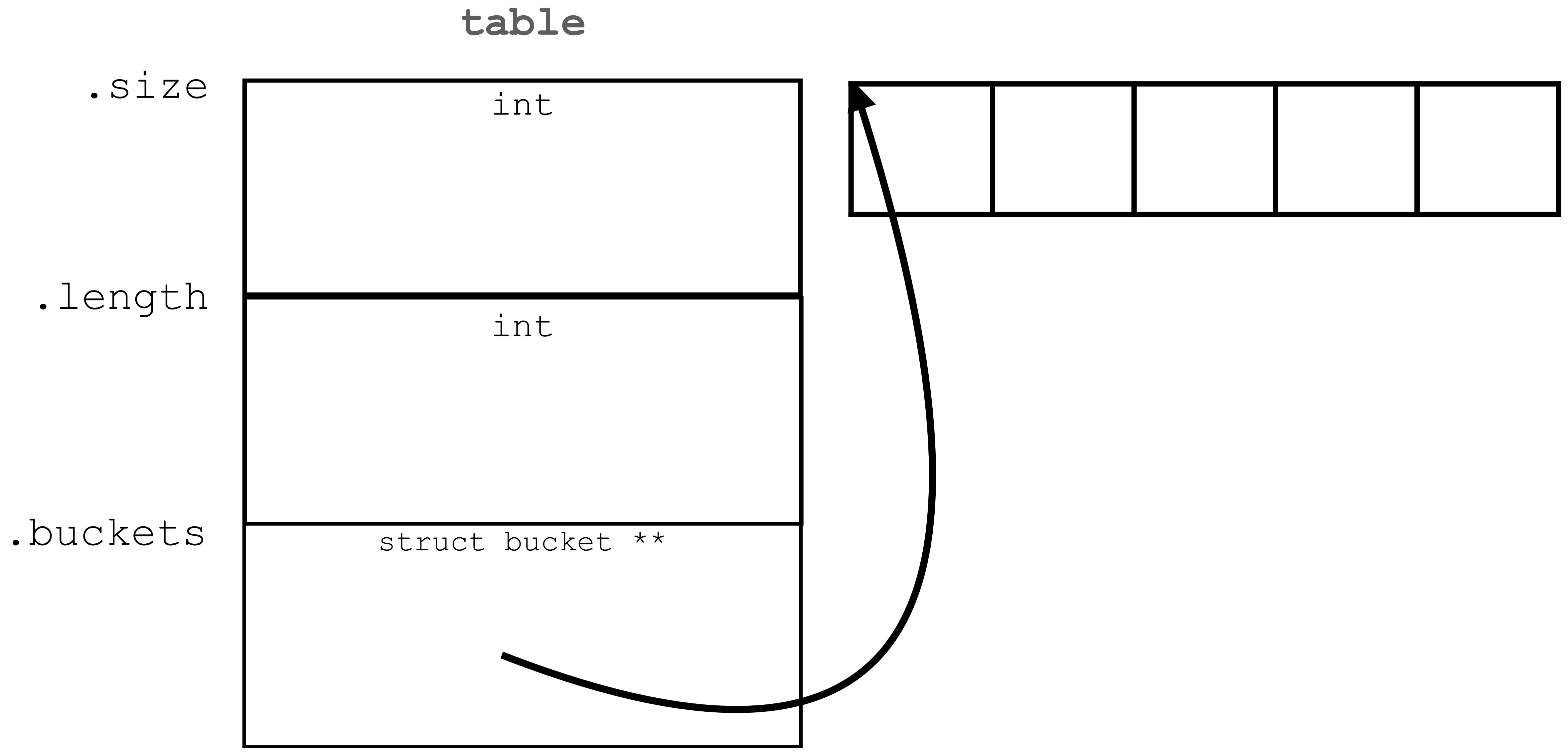
```
struct table {
        int size;
        int length;
        struct bucket *buckets[];
};
```



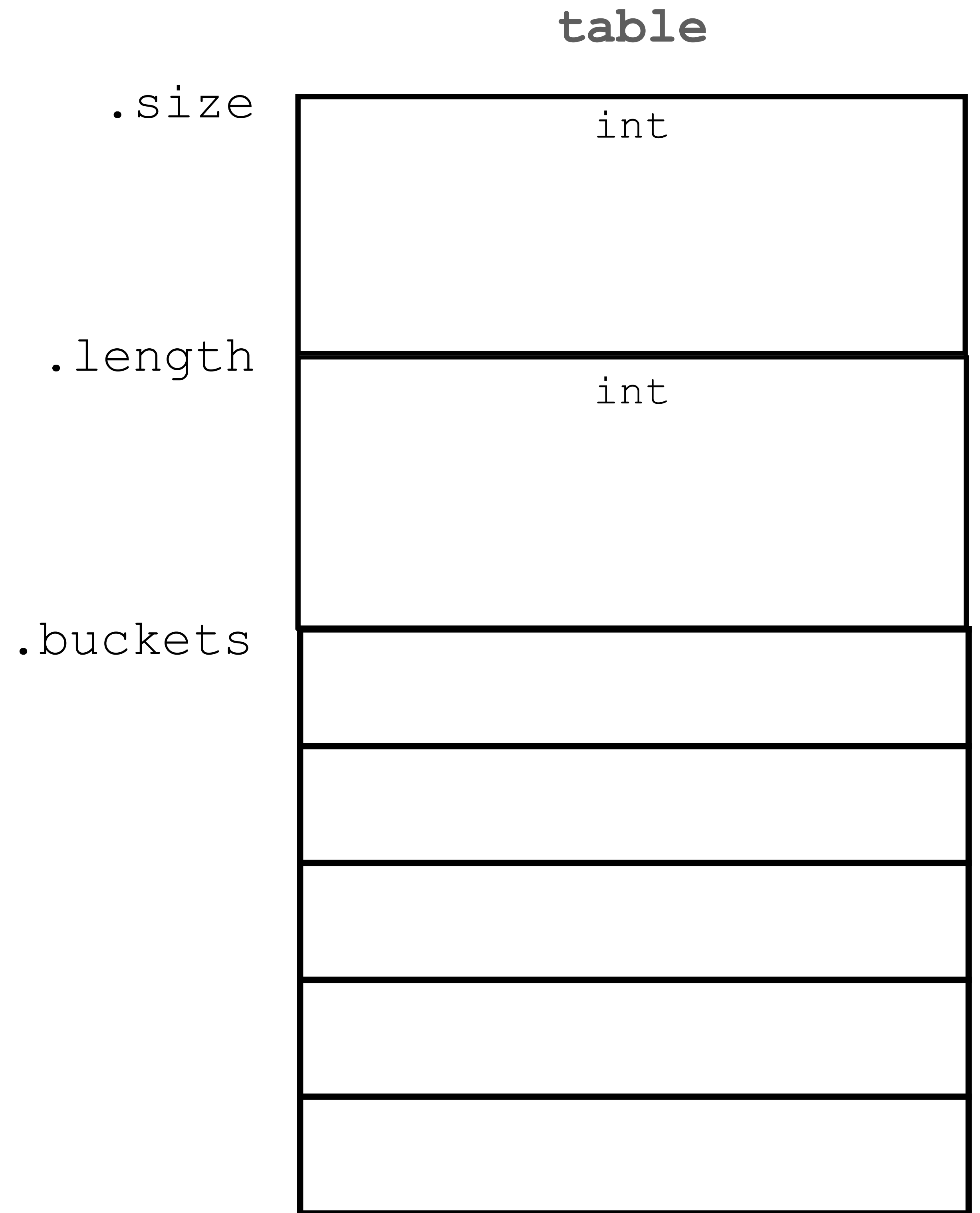
Interlude: Flexible Array Member

Memory Layout

```
struct table {
        int size;
        int length;
        struct bucket **buckets;
};
```



```
struct table {
        int size;
        int length;
        struct bucket *buckets[];
};
```



Interlude: Flexible Array Member

Allocation

```
struct table {
        int size;
        int length;
        struct bucket **buckets;
};


int size = 1024;

struct table *t =
  malloc(sizeof(struct table));

t->buckets =
  malloc(size * sizeof(struct bucket*));
```

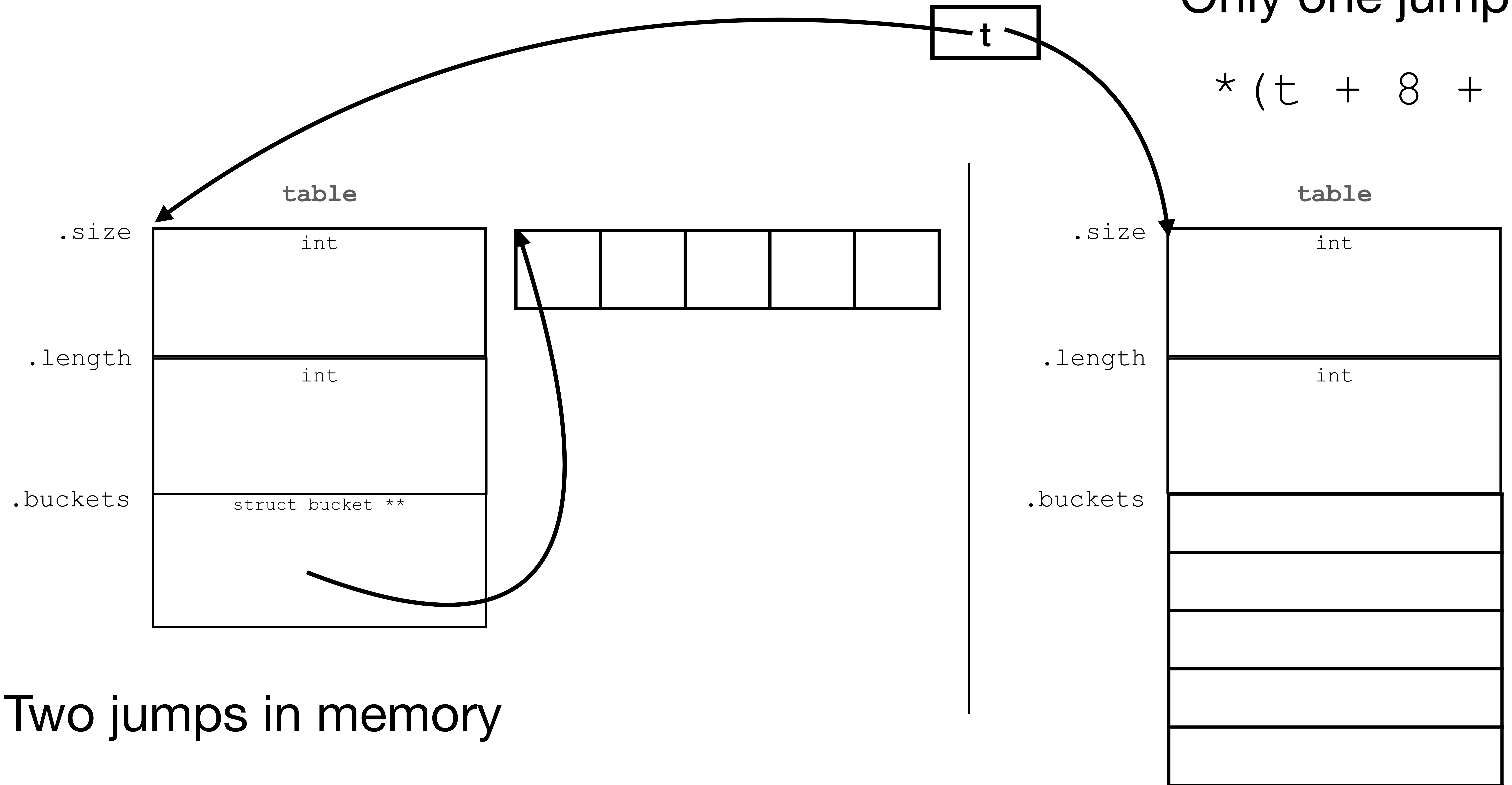
```
struct table {
        int size;
        int length;
        struct bucket *buckets[];
};


int size = 1024;

struct table *t =
  malloc(sizeof(struct table)
         + size * sizeof(struct bucket*));
```

Interlude: Flexible Array Member

Accessing `t->buckets[3];`



Only one jump!

`*(t + 8 + i * size)`

Two jumps in memory

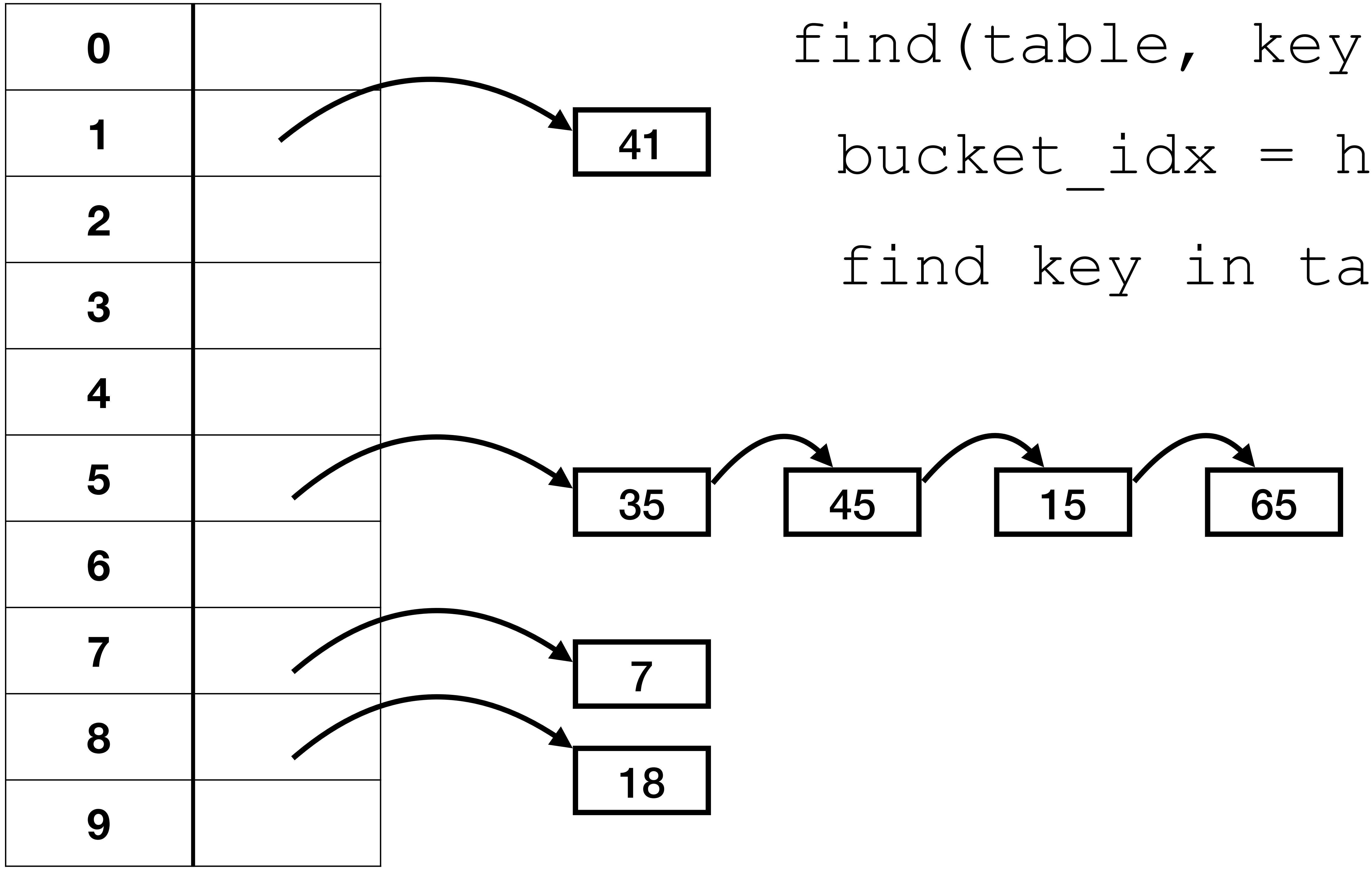
Interlude: Flexible Array Member

- The last element of a structure may have an incomplete array type (empty bracket)
- `sizeof` does not include the incomplete field
- `struct table *ptr = malloc(sizeof(struct table) + extra);`
- Slight performance boost

Chaining

Insert

- Each slot is a *list* of key-value pairs, called a *bucket*



```
find(table, key):
  bucket_idx = hash(key) % table->size
  find key in table->buckets[bucket_idx]:
```

Chaining

Time Complexity

- What is complexity for accessing elements?
  - $O(\text{length of the chain})$
- What is the length of the chain in the worst case?
- $O(n)$
  - This happens for a really bad hash function (e.g. $hash(k) = 1$)
- What if we have a good hash function (that has uniform distribution over a range of integers)?
  - What is the average (expected) length of a chain?
  - $O(\frac{\text{\#elements}}{\text{\#buckets}})$: this ratio is called *load factor*.

Chaining

Time Complexity

- In practice, hash tables are very fast
    - Typically faster than BSTs
- Especially we can keep the load factor $O(1)$
    - Analysis deferred to algorithms

Hash Table

Handling Collision

- Two approaches:
    1. ~~Chaining: put a list in each bucket~~
    2. Probing: use spare space in the array

Probing

- If the bucket is occupied, use the next one.

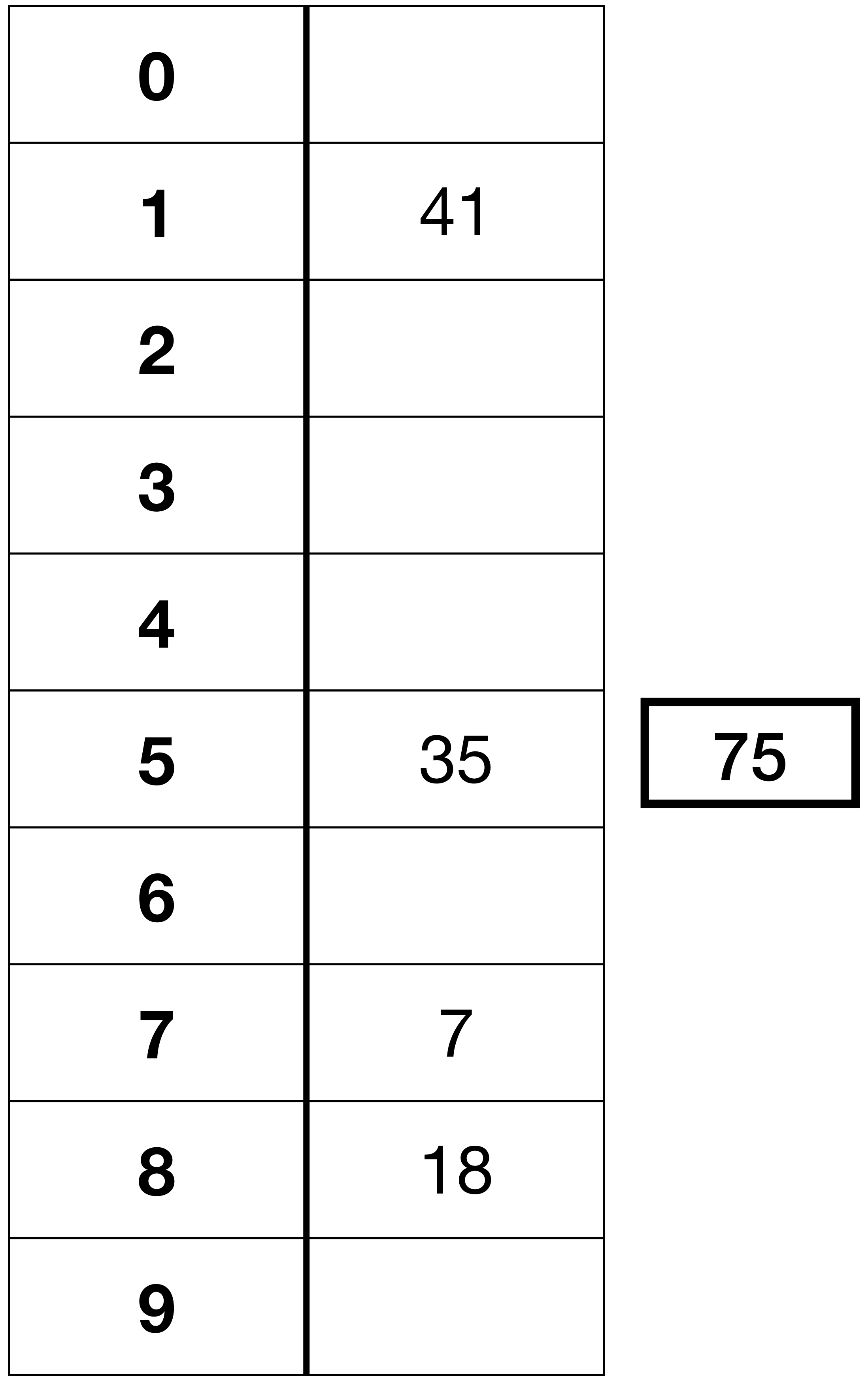
Probing

- If the bucket is occupied, use the next one.

0	
1	41
2	
3	
4	
5	35
6	
7	7
8	18
9	

Probing

- If the bucket is occupied, use the next one.



0	
1	41
2	
3	
4	
5	35
6	
7	7
8	18
9	

75

Probing

- If the bucket is occupied, use the next one.

0	
1	41
2	
3	
4	
5	35
6	75
7	7
8	18
9	

- Wrap around when reaching the end of array
- The table *must* have some extra space, i.e. load factor has to be ≤ 1
- Many flavors of "next one":
  - *Linear probing*: +1 at a time
  - *Quadratic probing*: * 2 at a time
  - ...

Probing

Linear probing (example)

```
struct bucket {
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

- insert("alice", 400)

Probing

Linear probing (example)

```
struct bucket {
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	
4	
5	("alice", 400)
6	
7	
8	
9	

- insert("bob", 30)

Probing

Linear probing (example)

```
struct bucket {
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	
5	("alice", 400)
6	
7	
8	
9	

- insert("carl", 50)

Probing

Linear probing (example)

```
struct bucket {
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	("alice", 400)
6	
7	
8	
9	

- insert("eve", 100)

Probing

Linear probing (example)

```
struct bucket {
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	("alice", 400)
6	("eve", 100)
7	("david", 60)
8	
9	

- insert("david", 60)

Probing

Linear probing (example)

```
struct bucket {
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	("alice", 400)
6	("eve", 100)
7	("david", 60)
8	
9	

- find("eve")
  - Go to 3 bucket
  - Move down until we find "eve" or until we hit empty bucket
- return 100

Probing

Linear probing (example)

```
struct bucket {
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	("alice", 400)
6	("eve", 100)
7	("david", 60)
8	
9	

- find("karl")
  - Go to 4 bucket
  - Move down until we find "karl" or until we hit empty bucket
- No "karl" in table

Probing

Linear probing (example)

```
struct bucket {
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	("alice", 400)
6	("eve", 100)
7	("david", 60)
8	
9	

- remove("alice")
  - Go to 5
  - Move down until we find "alice"

Probing

Linear probing (example)

```
struct bucket {
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	
6	("eve", 100)
7	("david", 60)
8	
9	

- remove("alice")
  - Go to 5
  - Move down until we find "alice"

Probing

Linear probing (example)

```
struct bucket {
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	
6	("eve", 100)
7	("david", 60)
8	
9	

- Find("eve")
  - Go to 3
  - How far do we move down?

Probing

Linear probing (example)

```
struct bucket {
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	
6	("eve", 100)
7	("david", 60)
8	
9	

- When we removed "alice" we left a hole
- When searching for "eve" if we stop at the hole, we won't find "eve"
- But if we don't stop at empty spots, we have to search through the entire array if a key doesn't exist

Probing

Linear probing (example)

```
struct bucket {
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	
6	("eve", 100)
7	("david", 60)
8	
9	

- A bucket can be in one of three states:
  - Occupied (key != NULL)
  - Empty, but was always empty
  - Empty, but previously occupied

Probing

Linear probing (example)

true when previously occupied

```
struct bucket {
        bool removed;
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	
6	("eve", 100)
7	("david", 60)
8	
9	

- A bucket can be in one of three states:
  - Occupied (key != NULL)
  - Empty, but was always empty
  - Empty, but previously occupied

Probing

Linear probing (example)

true when previously occupied

```
struct bucket {
        bool removed;
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	REMOVED
6	("eve", 100)
7	("david", 60)
8	
9	

- Find("eve")
  - Go to 3
  - Move down until we find "eve", or until we hit an empty, non-removed bucket

Probing

Linear probing (example)

true when
previously occupied

```
struct bucket {
        bool removed;
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	
6	("eve", 100)
7	("david", 60)
8	
9	

- Find("eve")
  - Go to 3
  - Move down until we find "eve", or until we hit an empty, non-removed bucket
- This empty but removed bucket is sometimes called a *tombstone*

Probing

Linear probing

true when previously occupied

```
struct bucket {
        bool removed;
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	RIP
6	("eve", 100)
7	("david", 60)
8	
9	

- Find/Remove:
  - Move down until first empty bucket
  - If tombstone is encountered, continue searching
- Insert:
  - Move down until first empty bucket
  - If tombstone is encountered, we can reuse that bucket
  - But to avoid inserting duplicate keys, we need to continue searching until an unremoved bucket

Probing

Linear probing

```
struct bucket {
        bool removed;
        void *key;
        void *value;
};
```

- This is why a good hash function spreads out outputs
- If the hash function maps similar inputs to similar outputs, e.g. strlen, we would get clusters in the hash table.
  - Really bad for probing
  - Clusters mean we need to go through more buckets

Probing

Time Complexity

```
struct bucket {
        bool removed;
        void *key;
        void *value;
};
```

- Chaining: worst $O(n)$, average $O(1)$
- What is the worst case complexity when using probing?
  - Insertion: $O(n)$
    - Worst case: all elements are in one cluster, need to go through all to find unfilled bucket
  - Get: $O(\text{table\_size})$
    - Worst case: all empty buckets are tombstones
- On average, the number of probes is at most $1/(1 - \text{load factor})$

Probing

Time Complexity (Appendix)

- Let A_i be the event that the ith probe is occupied.
    - $\Pr[A_1] = n/m$, assuming n elements and m slots
    - $\Pr[A_2] = (n-1)/(m-1)$, since $n-1$ elements and $m-1$ slots are remaining, assuming uniform hashing
    - $\Pr[A_1 \cap A_2 \cap \ldots \cap A_{i-1}] = \frac{n}{m} \cdot \frac{n-1}{m-1} \cdots \frac{n-i+2}{m-i+2} \leq \left(\frac{n}{m}\right)^{i-1}$ = load factor^{i-1}
- $E[\text{\#probes}] = \sum_{i=1}^{\infty} \Pr[A_1 \cap \ldots \cap A_{i-1}]$
- $\leq \sum_{i=1}^{\infty} \text{load factor}^{i-1}$
- $= \sum_{i=0}^{\infty} \text{load factor}^{i}$
- $= \frac{1}{1 - \text{load factor}}$
- E.g. if the table is half full, the average number of probes is 1 / (1 - 0.5) = 2

Load Factor

Notes

- Keep load factor $O(1)$ makes all operations $O(1)$
- Systems typically keep load factor around 0.7 to 0.75
    - This is determined through experimentation
    - Space vs. time trade-off
- What should we do when we hit the maximum load factor?
    - Increase the # of buckets
    - Can we just realloc? I.e. put the same elements in the same buckets after expansion?

Maps

Complexity

<table>
<tr><th rowspan="2"></th><th colspan="2"><code>lookup</code></th><th colspan="2"><code>insert</code></th><th colspan="2"><code>remove</code></th></tr>
<tr><th><code>average</code></th><th><code>worst</code></th><th><code>average</code></th><th><code>worst</code></th><th><code>average</code></th><th><code>worst</code></th></tr>
<tr><td>ArrayList</td><td colspan="2">O(n)</td><td>O(1)</td><td>O(n)</td><td colspan="2">O(1)</td></tr>
<tr><td>Linked List</td><td colspan="2">O(n)</td><td colspan="2">O(1)</td><td colspan="2">O(1)</td></tr>
<tr><td>ArrayList (sorted)</td><td colspan="2">O(log n)</td><td colspan="2">O(n)</td><td colspan="2">O(n)</td></tr>
<tr><td>Linked List (sorted)</td><td colspan="2">O(n)</td><td colspan="2">O(1)</td><td colspan="2">O(1)</td></tr>
<tr><td>BST</td><td>O(log n)</td><td>O(n)</td><td>O(log n)</td><td>O(n)</td><td>O(log n)</td><td>O(n)</td></tr>
<tr><td>Hash Table</td><td>O(1)</td><td>O(n)</td><td>O(1)</td><td>O(n)</td><td>O(1)</td><td>O(n)</td></tr>
</table>

Hash Table

Epilogue

- Hash tables are excellent at insertion, removal, and looking up. What operations are they bad at?
- Operations that involve comparisons:
  - find_min and find_max
  - range look up: give me 10 < key < 20
  - Better to use a heap or BST for these
- Operations that involve ordering, insert "front" and "back"
  - Hash tables have no notion of "order" -- in C++, hash tables are called unordered_map

Hash Table

In one slide

- Array access is $O(1)$.
- Using arbitrary keys as array indices:
  - Hash functions turn any values into an integer. Ideally, this should be uniform.
  - Compress function forces integers into [0, table_size).
- Handling Collision:
  - Chaining: put a list in each bucket
  - Probing: use spare space in the array
- Load factor: the expected number of elements to go through
  - #elements / #buckets
  - Chaining: load factor has no limit; probing: load factor at most 1
  - Adjusting #buckets to keep load factor (0.7 - 0.75) -- time/space trade-off

Data Structures

- Establishing structures on the heap:
  - Indices: contiguous
    - $O(1)$ random access
    - difficult to reorder and reallocate
  - Pointer: scattered
    - sequential access
    - easy to reorder and reallocate

	Indices	**Pointers**
List	Array List	Linked List
Map	Hash Table	BST