Bit Arithmetics

CS143: lecture 15

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Agenda

- Unsigned arithmetics
- Bit packing
- Negative Number

Unsigned Arithmetics

- unsigned means without sign (+/-). In C, `unsigned int x = 4`
- Why unsigned?
  - 32 bits means 2^32 choices
  - If we don't allow negative number, we have range [0, 2^32 - 1]
  - If we have negative numbers, the maximum has to be smaller.
- Most things that we care about are non-negative
  - counts, lengths, indices, dimensions...

Unsigned Arithmetics

```
unsigned int n = 42;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1010
+ 0000 0000 0000 0000 0000 0000 0000 0001
-----------------------------------------
                                      1
```

Unsigned Arithmetics

```
unsigned int n = 42;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1010
+ 0000 0000 0000 0000 0000 0000 0000 0001
-----------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1011
```

Unsigned Arithmetics

```
unsigned int n = 42;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1010
+ 0000 0000 0000 0000 0000 0000 0000 0001
-----------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1011
+ 0000 0000 0000 0000 0000 0000 0000 0001
-----------------------------------------
```

Unsigned Arithmetics

```
unsigned int n = 42;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1010
+ 0000 0000 0000 0000 0000 0000 0000 0001
-----------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1011
+ 0000 0000 0000 0000 0000 0000 0000 0001
                                      1
-----------------------------------------
                                      0
```

Unsigned Arithmetics

```
unsigned int n = 42;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1010
+ 0000 0000 0000 0000 0000 0000 0000 0001
-----------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1011
+ 0000 0000 0000 0000 0000 0000 0000 0001
                                    1 1
-----------------------------------------
                                    00
```

Unsigned Arithmetics

```
unsigned int n = 42;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1010
+ 0000 0000 0000 0000 0000 0000 0000 0001
-----------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1011
+ 0000 0000 0000 0000 0000 0000 0000 0001
                                    1 1
-----------------------------------------
                                     100
```

Unsigned Arithmetics

```
unsigned int n = 42;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1010
+ 0000 0000 0000 0000 0000 0000 0000 0001
-----------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1011
+ 0000 0000 0000 0000 0000 0000 0000 0001
                                    1 1
-----------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1100
```

Unsigned Arithmetics

```
unsigned int n = 42;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1010
+ 0000 0000 0000 0000 0000 0000 0000 0001
-----------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1011
+ 0000 0000 0000 0000 0000 0000 0000 0001
                                    1 1
-----------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1100
+ 0000 0000 0000 0000 0000 0000 0000 0001
-----------------------------------------
```

Unsigned Arithmetics

```
unsigned int n = 42;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
   0000 0000 0000 0000 0000 0000 0010 1010
 + 0000 0000 0000 0000 0000 0000 0000 0001
-----------------------------------------
   0000 0000 0000 0000 0000 0000 0010 1011
 + 0000 0000 0000 0000 0000 0000 0000 0001
                                     1 1
-----------------------------------------
   0000 0000 0000 0000 0000 0000 0010 1100
 + 0000 0000 0000 0000 0000 0000 0000 0001
-----------------------------------------
   0000 0000 0000 0000 0000 0000 0010 1101
```

Unsigned Arithmetics

```
unsigned int n = 42;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1010
+ 0000 0000 0000 0000 0000 0000 0000 0001
-----------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1011
+ 0000 0000 0000 0000 0000 0000 0000 0001
                                    1 1
-----------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1100
+ 0000 0000 0000 0000 0000 0000 0000 0001
-----------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1101
```

hex?

Unsigned Arithmetics

```
unsigned int n = 42;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
   0000 0000 0000 0000 0000 0000 0010 1010
 + 0000 0000 0000 0000 0000 0000 0000 0001
-----------------------------------------
   0000 0000 0000 0000 0000 0000 0010 1011
 + 0000 0000 0000 0000 0000 0000 0000 0001
                                     1 1
-----------------------------------------
   0000 0000 0000 0000 0000 0000 0010 1100
 + 0000 0000 0000 0000 0000 0000 0000 0001
-----------------------------------------
   0000 0000 0000 0000 0000 0000 0010 1101

                        hex?     0x0000002D
```

Unsigned Arithmetics

```
unsigned int n = 42;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
   0000 0000 0000 0000 0000 0000 0010 1010
 + 0000 0000 0000 0000 0000 0000 0000 0001
-----------------------------------------
   0000 0000 0000 0000 0000 0000 0010 1011
 + 0000 0000 0000 0000 0000 0000 0000 0001
                                      1 1
-----------------------------------------
   0000 0000 0000 0000 0000 0000 0010 1100
 + 0000 0000 0000 0000 0000 0000 0000 0001
-----------------------------------------
   0000 0000 0000 0000 0000 0000 0010 1101
```

hex? 0x00000002D

decimal?

Unsigned Arithmetics

```
unsigned int n = 42;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
   0000 0000 0000 0000 0000 0000 0010 1010
 + 0000 0000 0000 0000 0000 0000 0000 0001
-----------------------------------------
   0000 0000 0000 0000 0000 0000 0010 1011
 + 0000 0000 0000 0000 0000 0000 0000 0001
                                     1 1
-----------------------------------------
   0000 0000 0000 0000 0000 0000 0010 1100
 + 0000 0000 0000 0000 0000 0000 0000 0001
-----------------------------------------
   0000 0000 0000 0000 0000 0000 0010 1101

                          hex?     0x0000002D
                      decimal?            45
```

Unsigned Arithmetics

```
unsigned int n = UINT_MAX;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1111 1111 1111 1111 1111 1111 1111 1111
+ 0000 0000 0000 0000 0000 0000 0000 0001
                                        1
-----------------------------------------
                                        0
```

Unsigned Arithmetics

```
unsigned int n = UINT_MAX;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1111 1111 1111 1111 1111 1111 1111 1111
+ 0000 0000 0000 0000 0000 0000 0000 0001
                                     1 1
-----------------------------------------
                                     00
```

Unsigned Arithmetics

```
unsigned int n = UINT_MAX;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1111 1111 1111 1111 1111 1111 1111 1111
+ 0000 0000 0000 0000 0000 0000 0000 0001
                                   1 1 1
-----------------------------------------
                                   000
```

Unsigned Arithmetics

```
unsigned int n = UINT_MAX;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1111 1111 1111 1111 1111 1111 1111 1111
+ 0000 0000 0000 0000 0000 0000 0000 0001
                                1  1 1 1
-----------------------------------------
                                0000
```

Unsigned Arithmetics

```
unsigned int n = UINT_MAX;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1111 1111 1111 1111 1111 1111 1111 1111
+ 0000 0000 0000 0000 0000 0000 0000 0001
  1 1111 1111 1111 1111 1111 1111 1111 111
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0000 0000
```

Unsigned Arithmetics

```
unsigned int n = UINT_MAX;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1111 1111 1111 1111 1111 1111 1111 1111
+ 0000 0000 0000 0000 0000 0000 0000 0001
  1 1111 1111 1111 1111 1111 1111 1111 111
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0000 0000
```

- The highest carry is dropped when addition overflows.

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1101
- 0000 0000 0000 0000 0000 0000 0000 0001
-------------------------------------------
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1101
- 0000 0000 0000 0000 0000 0000 0000 0001
-------------------------------------------
                                        0
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1101
- 0000 0000 0000 0000 0000 0000 0000 0001
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1100
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1101
- 0000 0000 0000 0000 0000 0000 0000 0001
-----------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1100
- 0000 0000 0000 0000 0000 0000 0000 0001
-----------------------------------------
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1101
- 0000 0000 0000 0000 0000 0000 0000 0001
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1100
- 0000 0000 0000 0000 0000 0000 0000 0001
                                       1
-------------------------------------------
                                       1
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1101
- 0000 0000 0000 0000 0000 0000 0000 0001
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1100
- 0000 0000 0000 0000 0000 0000 0000 0001
                                       1
-------------------------------------------
                                       1
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1101
- 0000 0000 0000 0000 0000 0000 0000 0001
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1100
- 0000 0000 0000 0000 0000 0000 0000 0001
                                     1 1
-------------------------------------------
                                       11
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1101
- 0000 0000 0000 0000 0000 0000 0000 0001
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1100
- 0000 0000 0000 0000 0000 0000 0000 0001
                                     1 1
-------------------------------------------
                                      011
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1101
- 0000 0000 0000 0000 0000 0000 0000 0001
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1100
- 0000 0000 0000 0000 0000 0000 0000 0001
                                     1 1
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1011
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1101
- 0000 0000 0000 0000 0000 0000 0000 0001
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1100
- 0000 0000 0000 0000 0000 0000 0000 0001
                                      1 1
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1011
- 0000 0000 0000 0000 0000 0000 0000 0001
-------------------------------------------
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1101
- 0000 0000 0000 0000 0000 0000 0000 0001
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1100
- 0000 0000 0000 0000 0000 0000 0000 0001
                                     1 1
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1011
- 0000 0000 0000 0000 0000 0000 0000 0001
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1010
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1101
- 0000 0000 0000 0000 0000 0000 0000 0001
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1100
- 0000 0000 0000 0000 0000 0000 0000 0001
                                     1 1
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1011
- 0000 0000 0000 0000 0000 0000 0000 0001
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1010
```

hex?

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1101
- 0000 0000 0000 0000 0000 0000 0000 0001
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1100
- 0000 0000 0000 0000 0000 0000 0000 0001
                                     1 1
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1011
- 0000 0000 0000 0000 0000 0000 0000 0001
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1010

                        hex?     0x0000002A
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1101
- 0000 0000 0000 0000 0000 0000 0000 0001
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1100
- 0000 0000 0000 0000 0000 0000 0000 0001
                                    1 1
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1011
- 0000 0000 0000 0000 0000 0000 0000 0001
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1010

                        hex?     0x00000002A
                    decimal?
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1101
- 0000 0000 0000 0000 0000 0000 0000 0001
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1100
- 0000 0000 0000 0000 0000 0000 0000 0001
                                     1 1
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1011
- 0000 0000 0000 0000 0000 0000 0000 0001
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1010

                        hex?     0x0000002A
                    decimal?      32 + 8 + 2
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1101
- 0000 0000 0000 0000 0000 0000 0000 0001
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1100
- 0000 0000 0000 0000 0000 0000 0000 0001
                                     1 1
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1011
- 0000 0000 0000 0000 0000 0000 0000 0001
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1010

                      hex?     0x0000002A
                  decimal?      32 + 8 + 2
                               2 * 16 + 10
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1101
- 0000 0000 0000 0000 0000 0000 0000 0001
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1100
- 0000 0000 0000 0000 0000 0000 0000 0001
                                     1 1
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1011
- 0000 0000 0000 0000 0000 0000 0000 0001
-------------------------------------------
  0000 0000 0000 0000 0000 0000 0010 1010

                    hex?     0x0000002A
                decimal?      32 + 8 + 2
                             2 * 16 + 10
                                     42
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1101
*                                      ?
```

- Many clever ways to implement multiplication -- beyond the scope

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  0000 0000 0000 0000 0000 0000 0010 1101
*                                        2
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
   0000 0000 0000 0000 0000 0000 0010 1101
<<                                      1
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
    0000 0000 0000 0000 0000 0000 0010 1101
<<                                       1
-----------------------------------------------
    0000 0000 0000 0000 0000 0000 0101 1010
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
    0000 0000 0000 0000 0000 0000 0010 1101
<<                                     1
-----------------------------------------
    0000 0000 0000 0000 0000 0000 0101 1010   = 0x5A
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
    0000 0000 0000 0000 0000 0000 0010 1101
<<                                     1
-----------------------------------------
    0000 0000 0000 0000 0000 0000 0101 1010   = 0x5A  = 90
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
   0000 0000 0000 0000 0000 0000 0010 1101
<<                                      1
-----------------------------------------
   0000 0000 0000 0000 0000 0000 0101 1010   = 0x5A  = 90
<<                                     16
-----------------------------------------
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
    0000 0000 0000 0000 0000 0000 0010 1101
<<                                       1
-----------------------------------------------
    0000 0000 0000 0000 0000 0000 0101 1010   = 0x5A  = 90
<<                                      16
-----------------------------------------------
    0000 0000 0101 1010 0000 0000 0000 0000
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
   0000 0000 0000 0000 0000 0000 0010 1101
<<                                      1
-----------------------------------------
   0000 0000 0000 0000 0000 0000 0101 1010   = 0x5A  = 90
<<                                     16
-----------------------------------------
   0000 0000 0101 1010 0000 0000 0000 0000
<<                                     16
-----------------------------------------
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
   0000 0000 0000 0000 0000 0000 0010 1101
<<                                      1
-----------------------------------------
   0000 0000 0000 0000 0000 0000 0101 1010   = 0x5A  = 90
<<                                     16
-----------------------------------------
   0000 0000 0101 1010 0000 0000 0000 0000
<<                                     16
-----------------------------------------
   0000 0000 0000 0000 0000 0000 0000 0000
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
   0000 0000 0000 0000 0000 0000 0010 1101
<<                                      1
-----------------------------------------
   0000 0000 0000 0000 0000 0000 0101 1010   = 0x5A  = 90
<<                                     16
-----------------------------------------
   0000 0000 0101 1010 0000 0000 0000 0000
<<                                     16
-----------------------------------------
   0000 0000 0000 0000 0000 0000 0000 0000
```

- When shift overflows, the bit is lost.

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
    0000 0000 0000 0000 0000 0000 0010 1101
>>                                       1
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
    0000 0000 0000 0000 0000 0000 0010 1101
>>                                       1
-----------------------------------------------
    0000 0000 0000 0000 0000 0000 0001 0110
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
    0000 0000 0000 0000 0000 0000 0010 1101
>>                                       1
-----------------------------------------------
    0000 0000 0000 0000 0000 0000 0001 0110   = 0x16
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
    0000 0000 0000 0000 0000 0000 0010 1101
>>                                       1
-----------------------------------------------
    0000 0000 0000 0000 0000 0000 0001 0110   = 0x16  = 22
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
    0000 0000 0000 0000 0000 0000 0010 1101
>>                                       1
-----------------------------------------------
    0000 0000 0000 0000 0000 0000 0001 0110   = 0x16  = 22
>>                                       8
-----------------------------------------------
    0000 0000 0000 0000 0000 0000 0000 0000
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
    0000 0000 0000 0000 0000 0000 0010 1101
>>                                       1
-----------------------------------------
    0000 0000 0000 0000 0000 0000 0001 0110   = 0x16  = 22
>>                                       8
-----------------------------------------
    0000 0000 0000 0000 0000 0000 0000 0000
```

- When shift underflows, the bit is lost.

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1101 0011 1100 0001 0010 0100 1001 1111
& 0101 0101 0101 0101 0101 0101 0101 0101
-----------------------------------------
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1101 0011 1100 0001 0010 0100 1001 1111
& 0101 0101 0101 0101 0101 0101 0101 0101
-----------------------------------------
                                      0001
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1101 0011 1100 0001 0010 0100 1001 1111
& 0101 0101 0101 0101 0101 0101 0101 0101
-----------------------------------------
                                     0101
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1101 0011 1100 0001 0010 0100 1001 1111
& 0101 0101 0101 0101 0101 0101 0101 0101
-----------------------------------------
                                    0101
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1101 0011 1100 0001 0010 0100 1001 1111
& 0101 0101 0101 0101 0101 0101 0101 0101
-----------------------------------------
                                   0111
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1101 0011 1100 0001 0010 0100 1001 1111
& 0101 0101 0101 0101 0101 0101 0101 0101
-----------------------------------------
                                1  0111
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1101 0011 1100 0001 0010 0100 1001 1111
& 0101 0101 0101 0101 0101 0101 0101 0101
-----------------------------------------
                              01 0111
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1101 0011 1100 0001 0010 0100 1001 1111
& 0101 0101 0101 0101 0101 0101 0101 0101
-----------------------------------------
  0101 0001 0100 0001 0000 0100 0001 0111
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1101 0011 1100 0001 0010 0100 1001 1111
| 0101 0101 0101 0101 0101 0101 0101 0101
-----------------------------------------
                                       1
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1101 0011 1100 0001 0010 0100 1001 1111
| 0101 0101 0101 0101 0101 0101 0101 0101
-----------------------------------------
                                     11
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

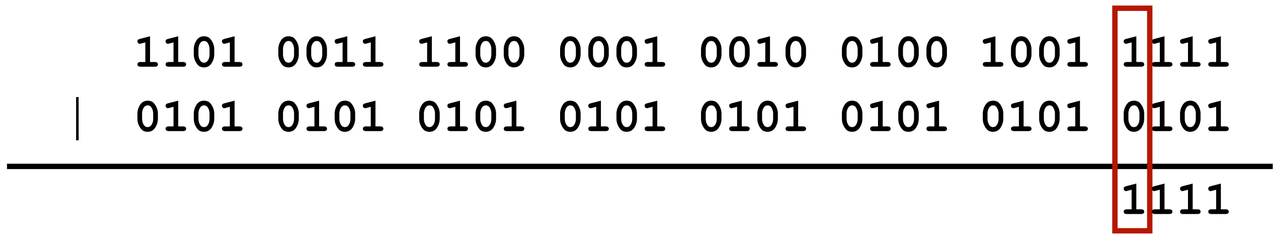
n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1101 0011 1100 0001 0010 0100 1001 1111
| 0101 0101 0101 0101 0101 0101 0101 0101
-----------------------------------------
                                     111
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

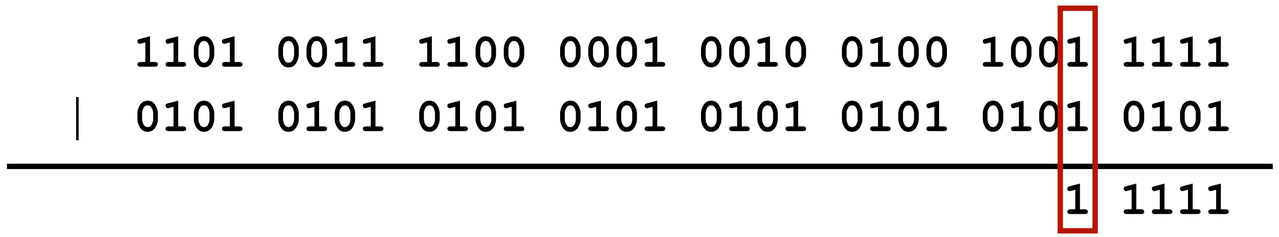
n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128



Unsigned Arithmetics

```
unsigned int n = 45;
```

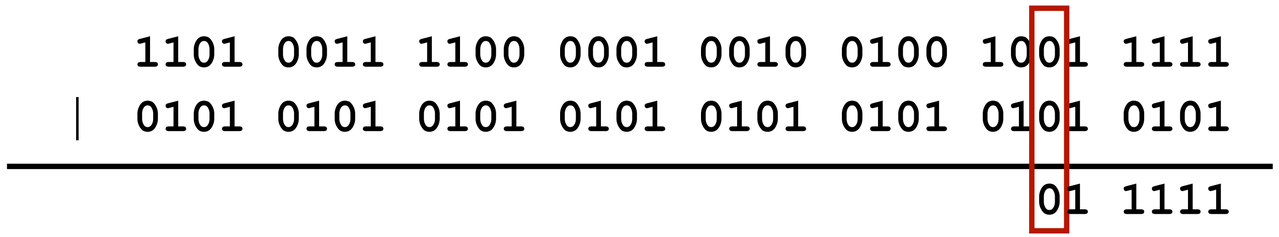
n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128



Unsigned Arithmetics

```
unsigned int n = 45;
```

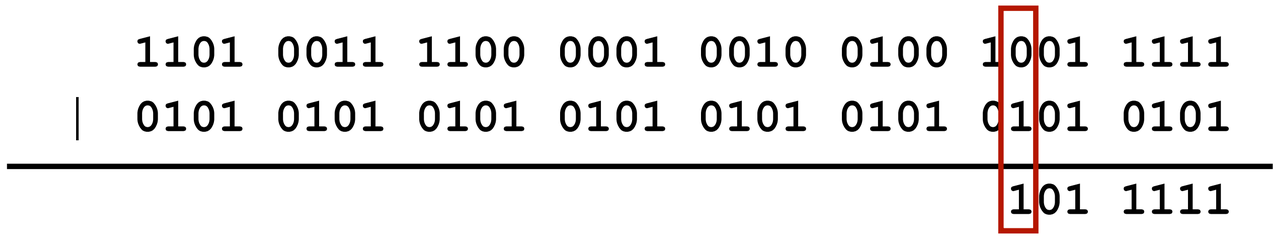
n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128



Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128



Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1101 0011 1100 0001 0010 0100 1001 1111
| 0101 0101 0101 0101 0101 0101 0101 0101
-----------------------------------------
  1101 0111 1101 0101 0111 0101 1101 1111
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
~ 1101 0011 1100 0001 0010 0100 1001 1111
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  ~ 1101 0011 1100 0001 0010 0100 1001 1111
-------------------------------------------
                                      0000
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  ~ 1101 0011 1100 0001 0010 0100 1001 1111
-------------------------------------------
                                  0110 0000
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  ~ 1101 0011 1100 0001 0010 0100 1001 1111
-------------------------------------------
                        1011 0110 0000
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  ~ 1101 0011 1100 0001 0010 0100 1001 1111
-------------------------------------------
    0010 1100 0011 1110 1101 1011 0110 0000
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

Exclusive-Or (XOR): 1 if the bits are different 0 if the bits are the same.

```
  1101 0011 1100 0001 0010 0100 1001 1111
^ 0101 0101 0101 0101 0101 0101 0101 0101
-----------------------------------------
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1101 0011 1100 0001 0010 0100 1001 1111
^ 0101 0101 0101 0101 0101 0101 0101 0101
-----------------------------------------
                                       0
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1101 0011 1100 0001 0010 0100 1001 1111
^ 0101 0101 0101 0101 0101 0101 0101 0101
-----------------------------------------
                                      10
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1101 0011 1100 0001 0010 0100 1001 1111
^ 0101 0101 0101 0101 0101 0101 0101 0101
-----------------------------------------
                                     010
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1101 0011 1100 0001 0010 0100 1001 1111
^ 0101 0101 0101 0101 0101 0101 0101 0101
-----------------------------------------
                                   1010
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1101 0011 1100 0001 0010 0100 1001 1111
^ 0101 0101 0101 0101 0101 0101 0101 0101
-----------------------------------------
                                0  1010
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1101 0011 1100 0001 0010 0100 1001 1111
^ 0101 0101 0101 0101 0101 0101 0101 0101
-----------------------------------------
                                10 1010
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1101 0011 1100 0001 0010 0100 1001 1111
^ 0101 0101 0101 0101 0101 0101 0101 0101
-----------------------------------------
                               100 1010
```

Unsigned Arithmetics

```
unsigned int n = 45;
```

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

```
  1101 0011 1100 0001 0010 0100 1001 1111
^ 0101 0101 0101 0101 0101 0101 0101 0101
-----------------------------------------
  1000 0110 1001 0100 0111 0001 1100 1010
```

Unsigned Arithmetics

- `+,-,<<,>>,&,|,~,^`
- `~` is not the same as `!`, `&` is not the same as `&&`, `|` is not the same as `||`
  - `!`, `&&`, `||` are *Boolean* operators -- they treat zero as false and non-zero as true
  - `!`, `&`, `|` are *bitwise* operators -- they operate on each bit.

Bit-packing

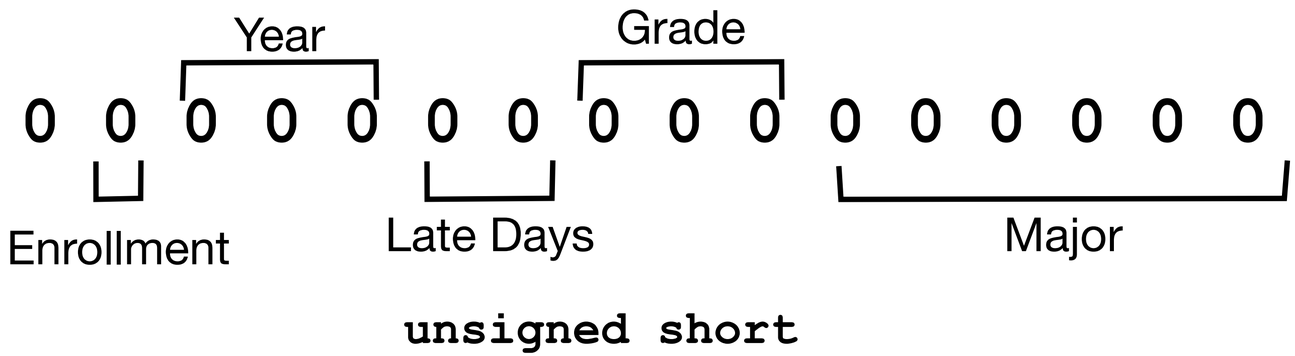
```
struct student {
        int is_enrolled;     /* is student formally enrolled in this class? */
        int year;
        int late_days_used;
        char letter_grade;
        char major[32];
};
```

Bit-packing

- Enrollment: 2 choices (1 bit)
- Year: 5 choices [1, 2, 3, 4, beyond-4] (3 bits)
- Late Days: 4 choices (2 bits)
- Letter grade: 5 choices (3 bits)
- Major: 53 majors (6 bits)
- None of these need 32 bit integers!
- 1 + 3 + 2 + 3 + 6 = 15, in fact we can fit the entire record inside a short.

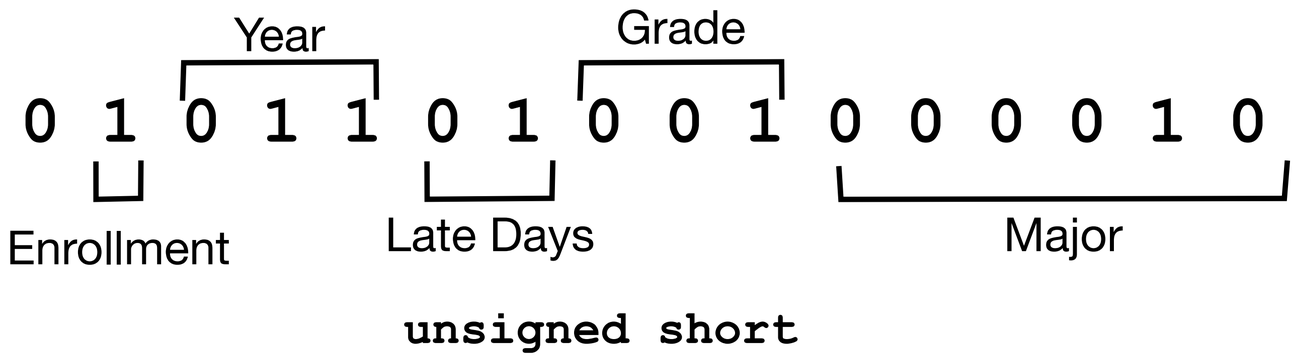
```
struct student {
        int is_enrolled;
        int year;
        int late_days_used;
        char letter_grade;
        char major[32];
};
```

Bit-packing



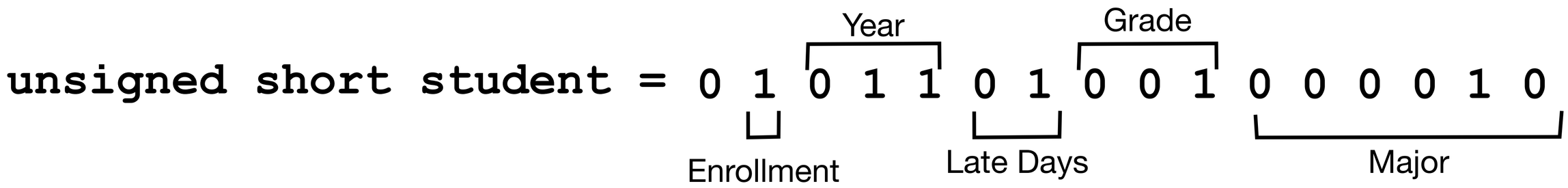
- Enrollment: 2 choices (1 bit)
- Year: 6 choices [0, 1, 2, 3, 4, beyond-4] (3 bits)
- Late Days: 4 choices (2 bits)
- Letter grade: 5 choices (3 bits)
- Major: 53 majors (6 bits)

Bit-packing



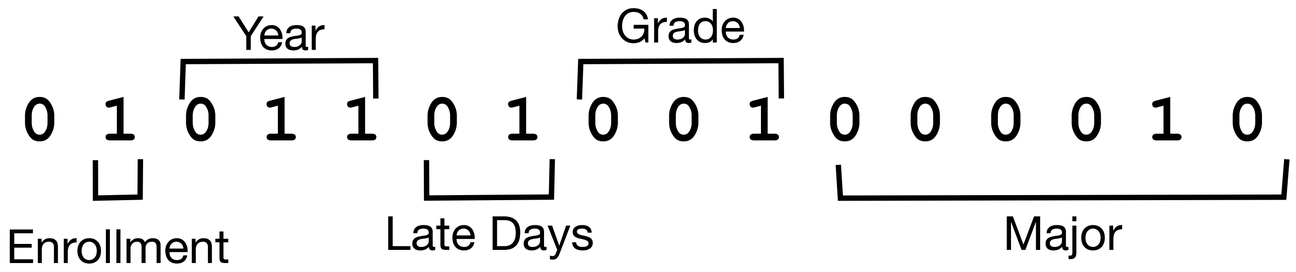
- Enrolled
- Year 3
- 1 late day
- 1st choice of grade
- 2nd choice of major

Bit-packing


unsigned short student = 0 1 0 1 1 0 1 0 0 1 0 0 0 0 1 0
Year
Grade
Enrollment
Late Days
Major

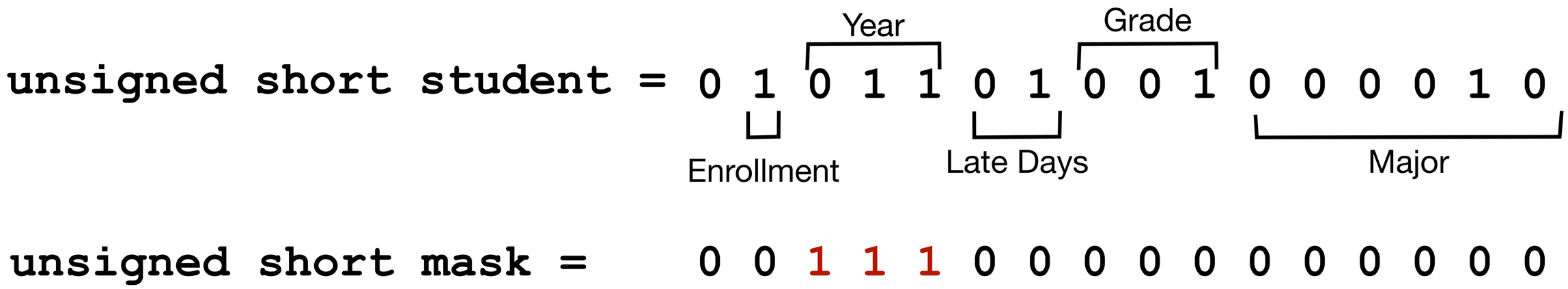
Bit-packing

```
unsigned short student = 0 1 0 1 1 0 1 0 0 1 0 0 0 0 1 0
```

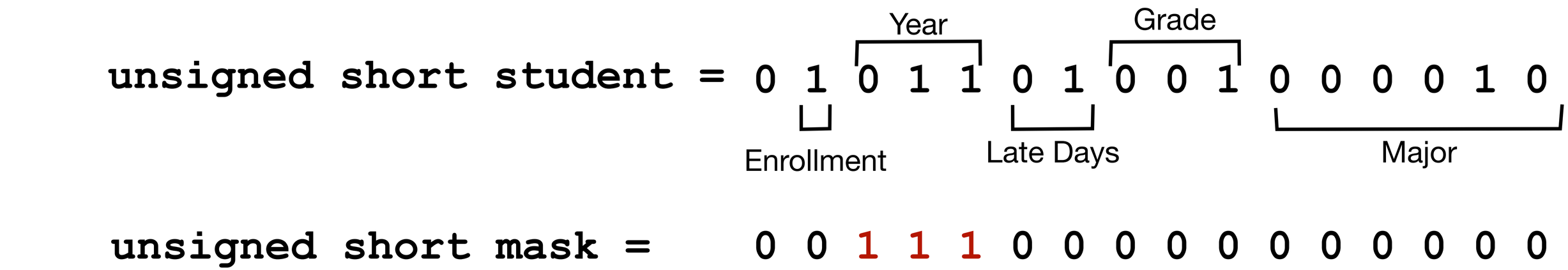



```
unsigned short mask =
```

Bit-packing

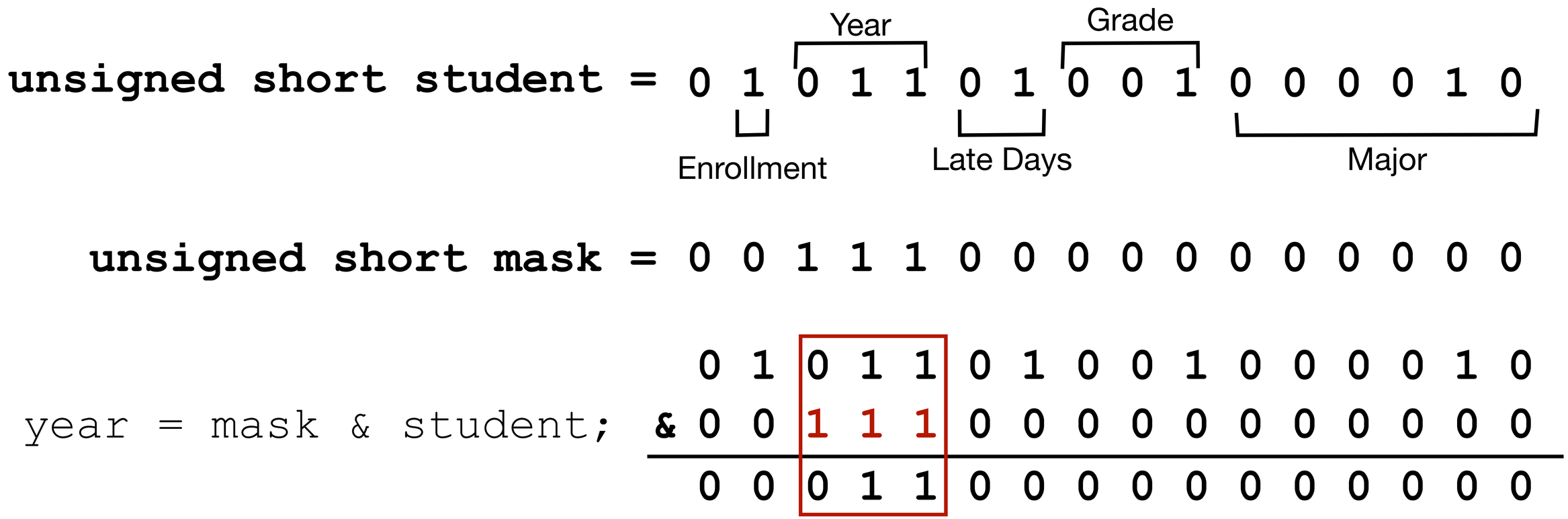

unsigned short student = 0 1 0 1 1 0 1 0 0 1 0 0 0 0 1 0
Year
Grade
Enrollment
Late Days
Major
unsigned short mask = 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0

Bit-packing

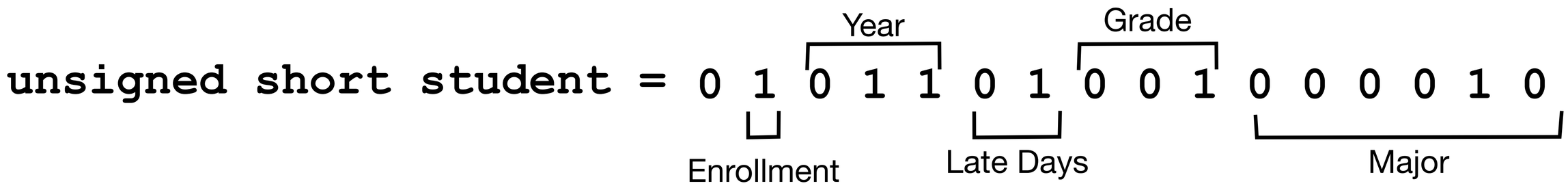



```
int year = mask & student;
```

Bit-packing


unsigned short student = 0 1 0 1 1 0 1 0 0 1 0 0 0 0 1 0
Year
Grade
Enrollment
Late Days
Major
unsigned short mask = 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0
int year = mask & student;
0 1 0 1 1 0 1 0 0 1 0 0 0 0 1 0
& 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0

Bit-packing



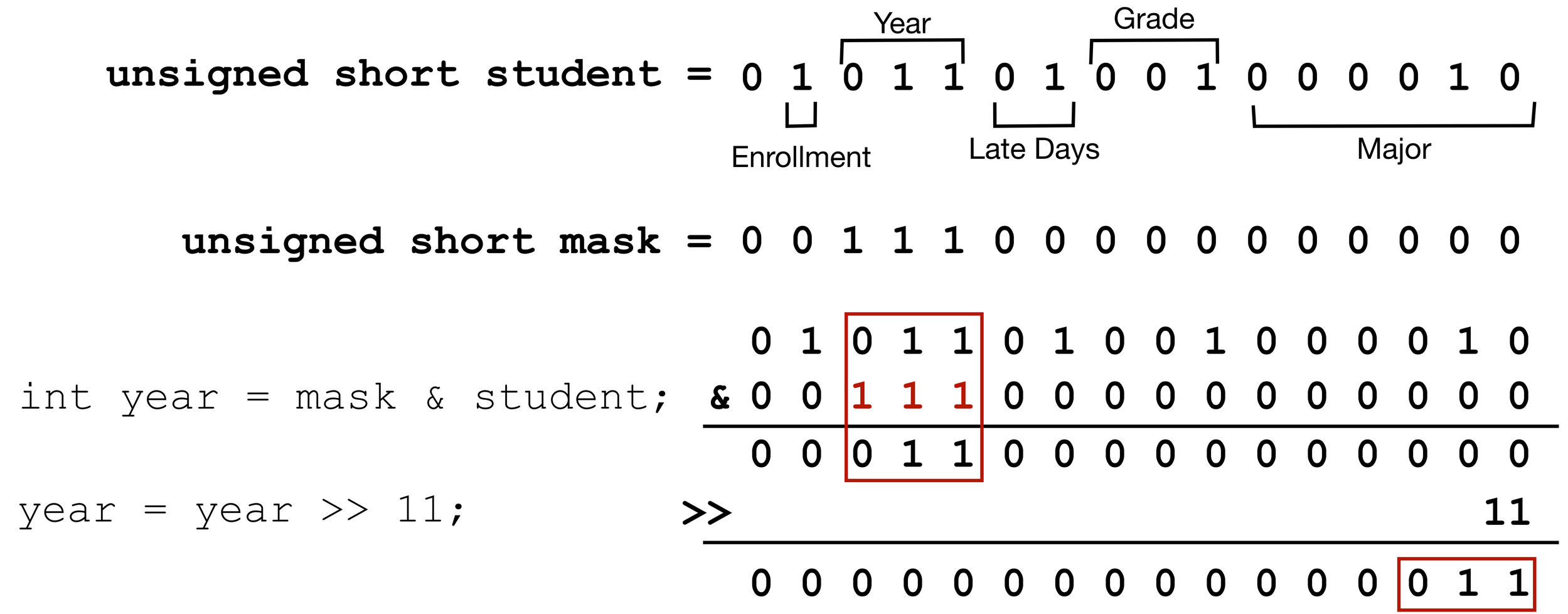
unsigned short mask = 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0

```
int year = mask & student;
```

```
  0 1 0 1 1 0 1 0 0 1 0 0 0 0 1 0
& 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0
-------------------------------
  0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0
```

```
year = year >> 11;
```

Bit-packing

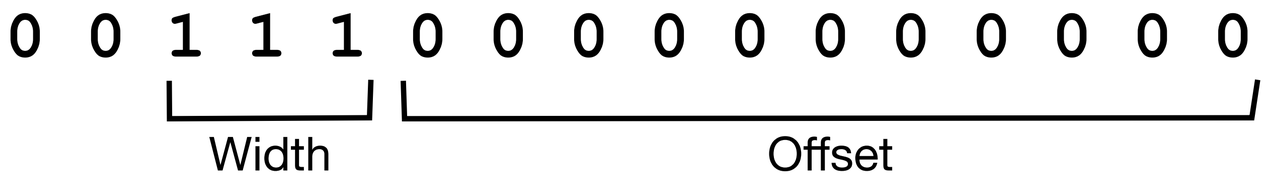

unsigned short student = 0 1 0 1 1 0 1 0 0 1 0 0 0 0 1 0
Year
Grade
Enrollment
Late Days
Major
unsigned short mask = 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0
0 1 0 1 1 0 1 0 0 1 0 0 0 0 1 0
int year = mask & student; & 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0
year = year >> 11; >> 11
0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1

Bit-packing

0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0

```
unsigned short mask = 14336;

unsigned short mask = 0x7 << 11;
```

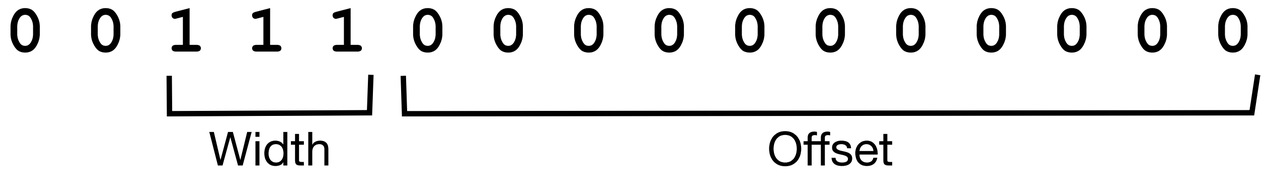
Bit-packing




```
unsigned short mask = 14336;

unsigned short mask = 0x7 << 11;


unsigned short mask = ~0u >> (16 - width) << offset;
```

Future Note: we need to write 0u instead of 0 because by default, 0 is signed, so >> is going to perform a signed right shift.

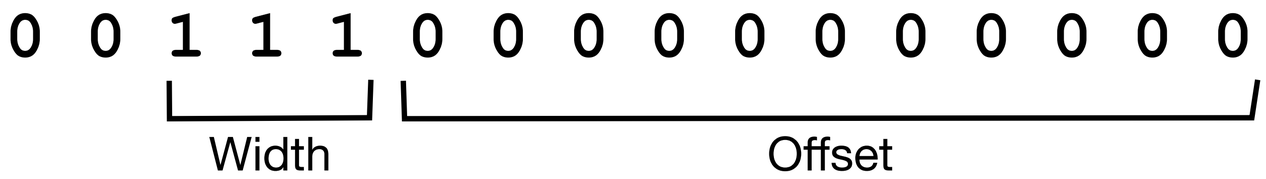
Bit-packing




```
unsigned short mask = ~0u >> (16 - width) << offset;
```

~ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

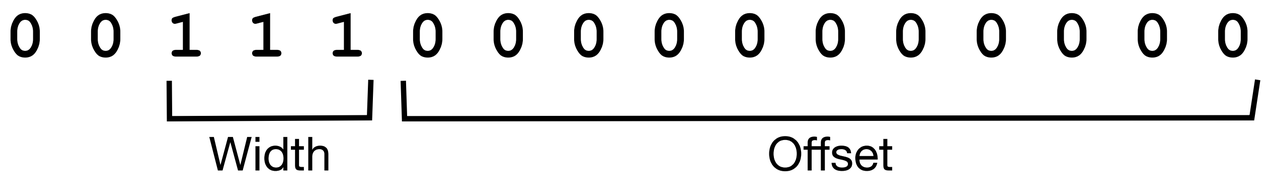
Bit-packing




```
unsigned short mask = ~0u >> (16 - width) << offset;
```

```
~ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
-------------------------------
  1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
```

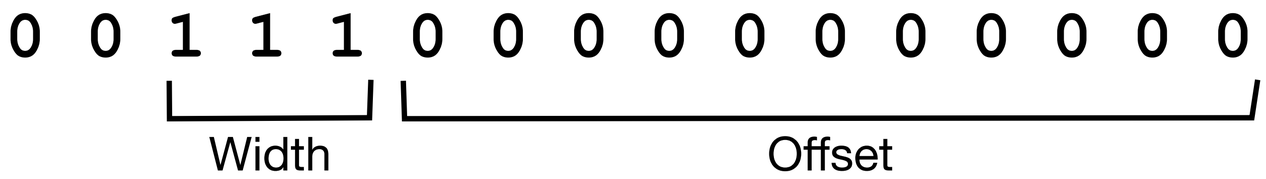
Bit-packing




```
unsigned short mask = ~0u >> (16 - width) << offset;
```

```
  ~ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  ---------------------------------
    1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
>>                                13
  ---------------------------------
```

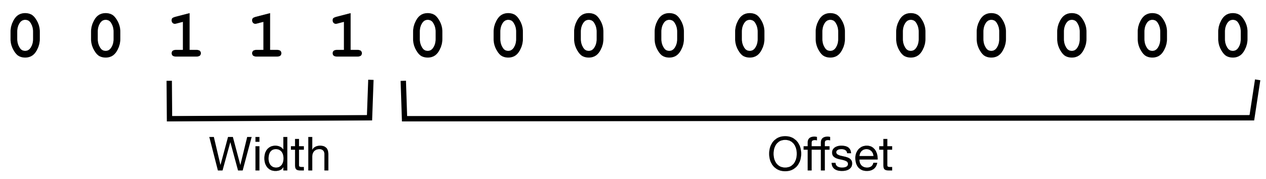
Bit-packing




```
unsigned short mask = ~0u >> (16 - width) << offset;
```

```
  ~ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  ---------------------------------
    1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
>>                                13
  ---------------------------------
    0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1
```

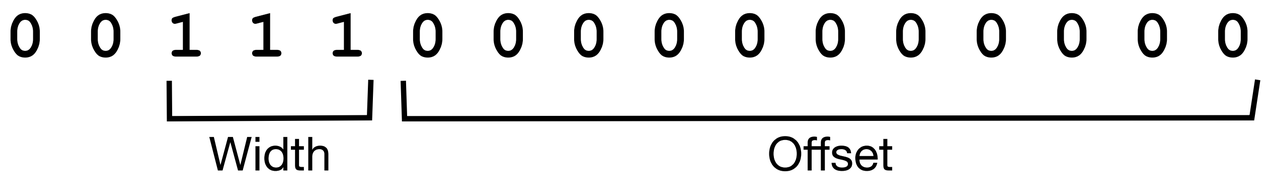
Bit-packing




```
unsigned short mask = ~0u >> (16 - width) << offset;
```

```
  ~ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  ---------------------------------
    1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
>>                                13
  ---------------------------------
    0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1
<<                                11
  ---------------------------------
```

Bit-packing



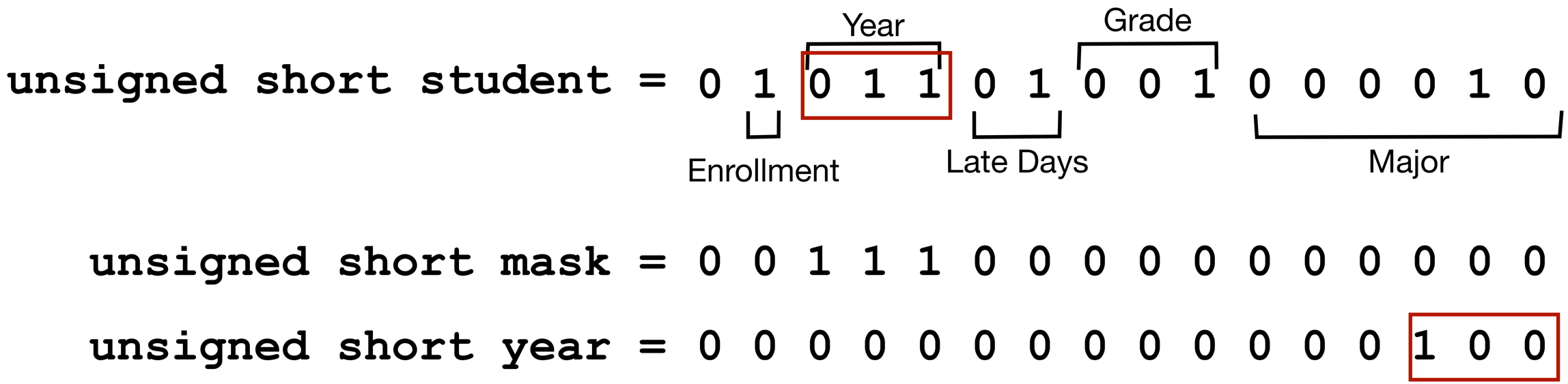

```
unsigned short mask = ~0u >> (16 - width) << offset;
```

```
  ~ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  -------------------------------
    1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
>>                             13
  -------------------------------
    0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1
<<                             11
  -------------------------------
    0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0
```

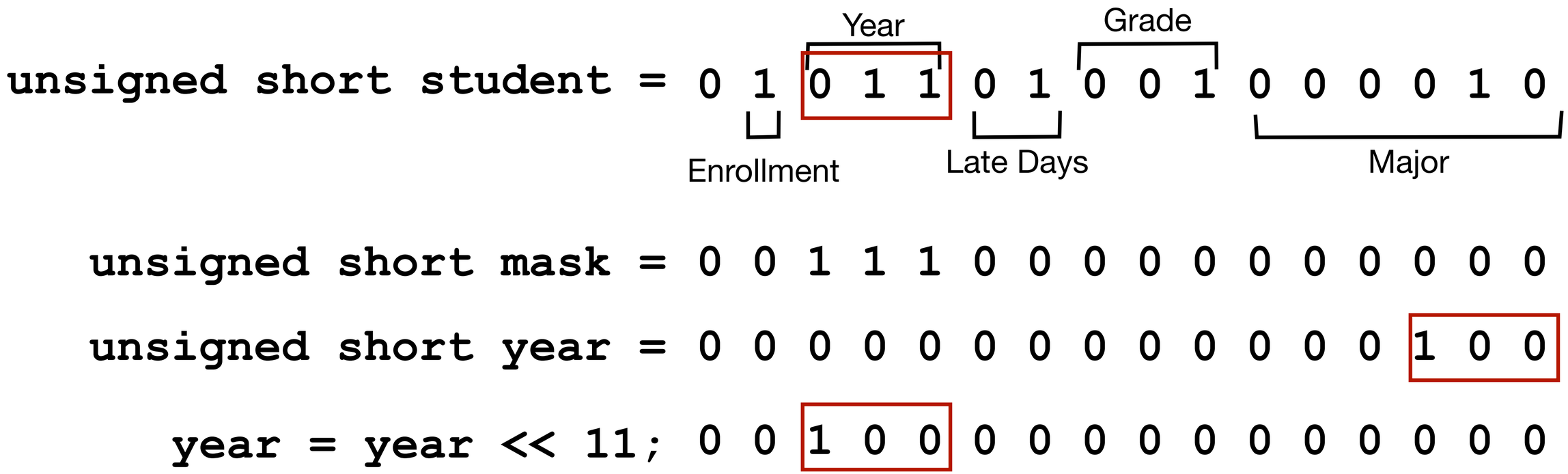
Bit-packing

```
unsigned short student = ...;
unsigned short year_mask = ~0u >> (16 - 3) << 11;

unsigned int year = (student & year_mask) >> 11;
```

Bit-packing


unsigned short student = 0 1 0 1 1 0 1 0 0 1 0 0 0 0 1 0
Year
Grade
Enrollment
Late Days
Major
unsigned short mask = 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0
unsigned short year = 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0

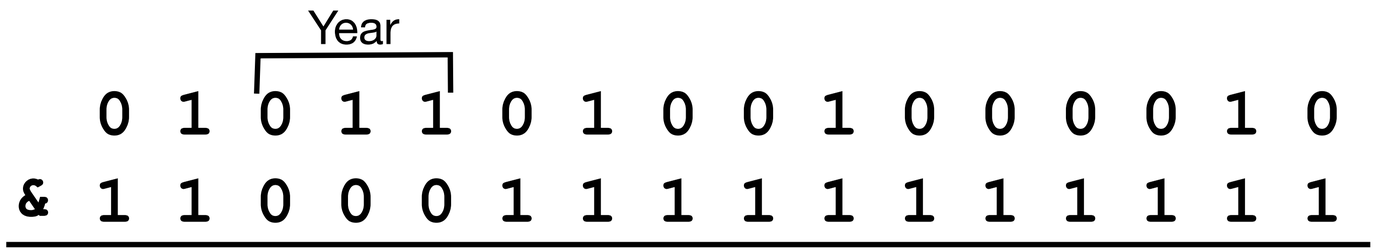
Bit-packing


unsigned short student = 0 1 0 1 1 0 1 0 0 1 0 0 0 0 1 0
Year
Grade
Enrollment
Late Days
Major
unsigned short mask = 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0
unsigned short year = 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
year = year << 11; 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0

??

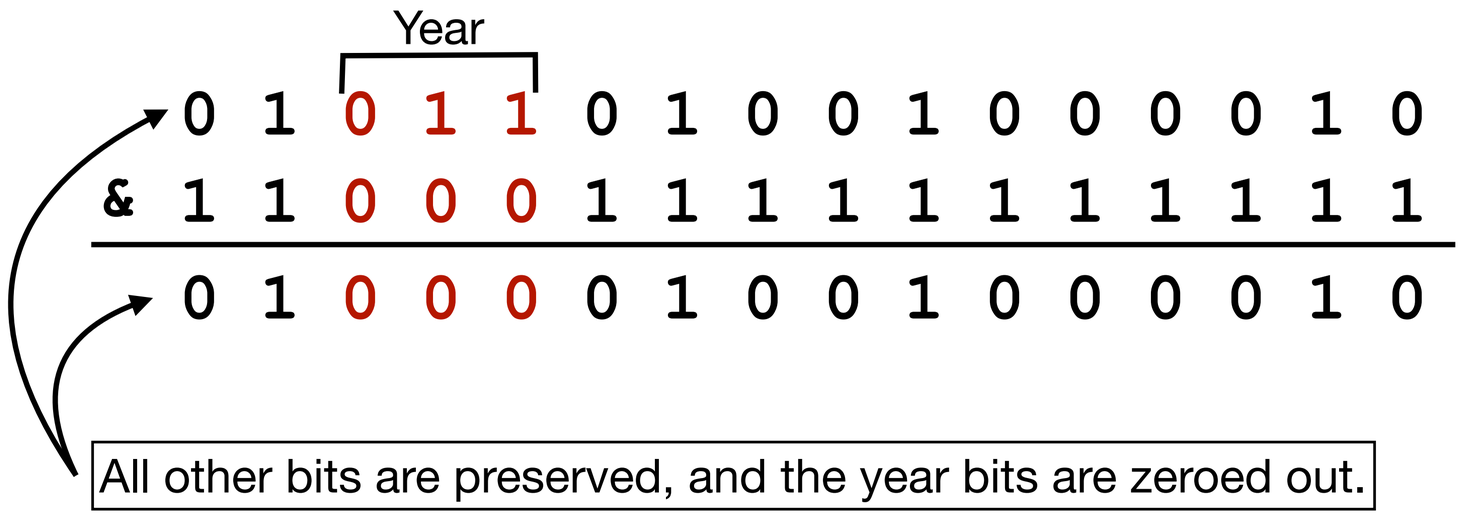
Bit-packing

```
student = student & ~year_mask;
```



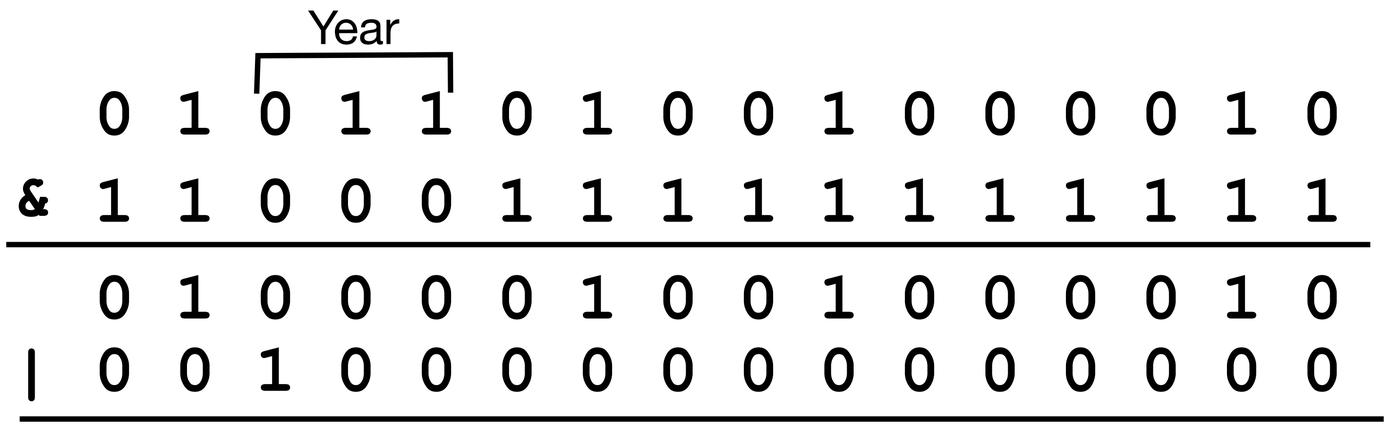
Bit-packing

```
student = student & ~year_mask;
```



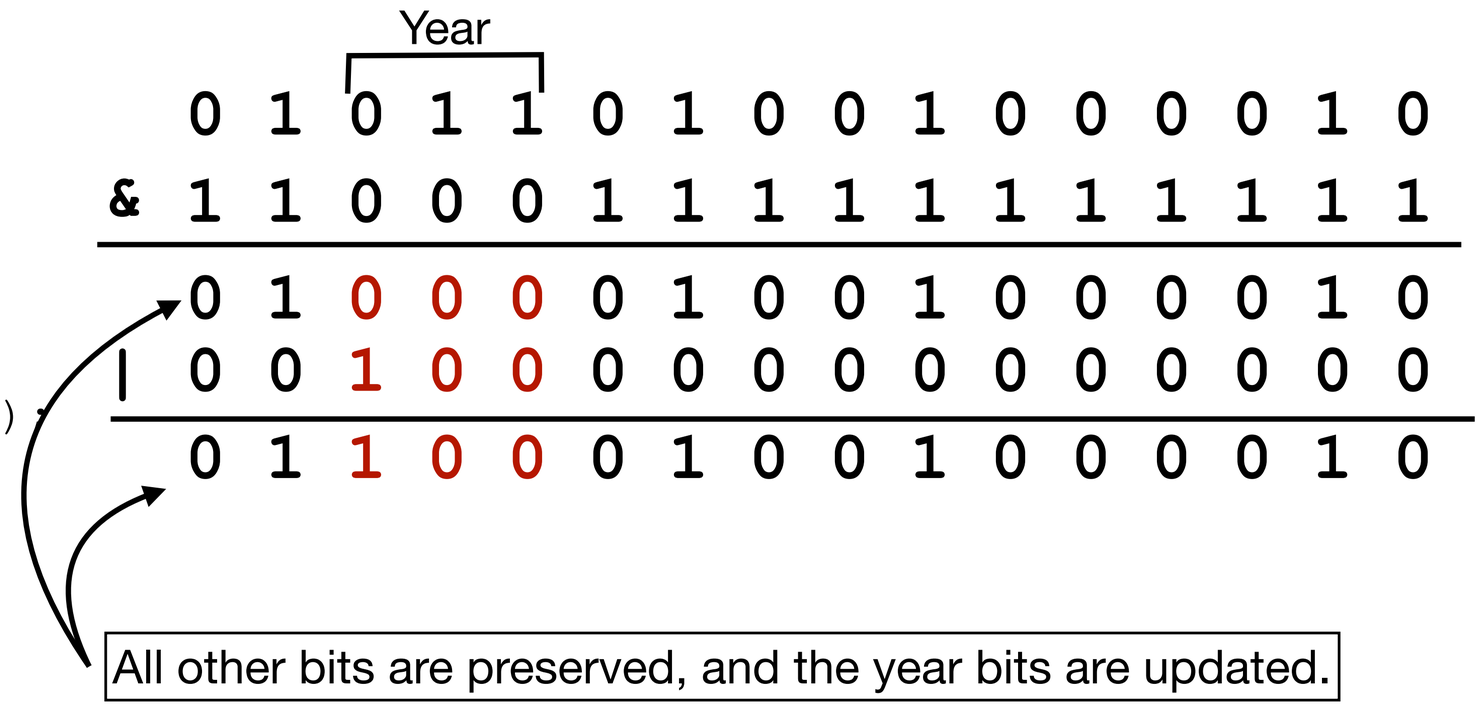
Bit-packing

```
student = student & ~year_mask;

student = student | (year << 11);
```



Bit-packing

```
student = student & ~year_mask;

student = student | (year << 11);
```



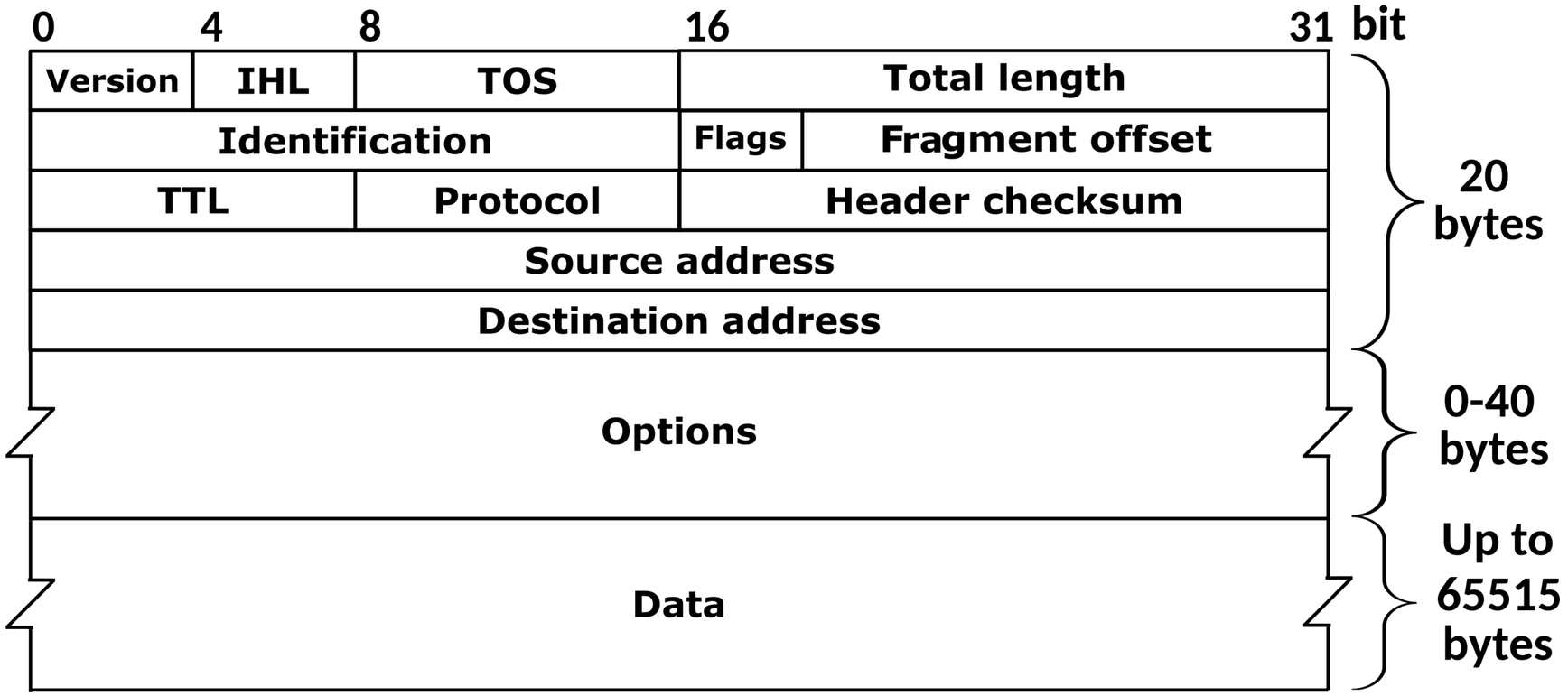
Bit-packing

Summary

- A mask has the bit fields of interest set 1 and all others 0.
- `& mask` gets the bits
- `& ~mask` clears out the bits
- `~0u >> (16 - width) << offset` creates a 16-bit mask with `width` and `offset`.
- After the bits are zeroed out, `|` is used to write new bits in the field.
- Does anyone use this irl?

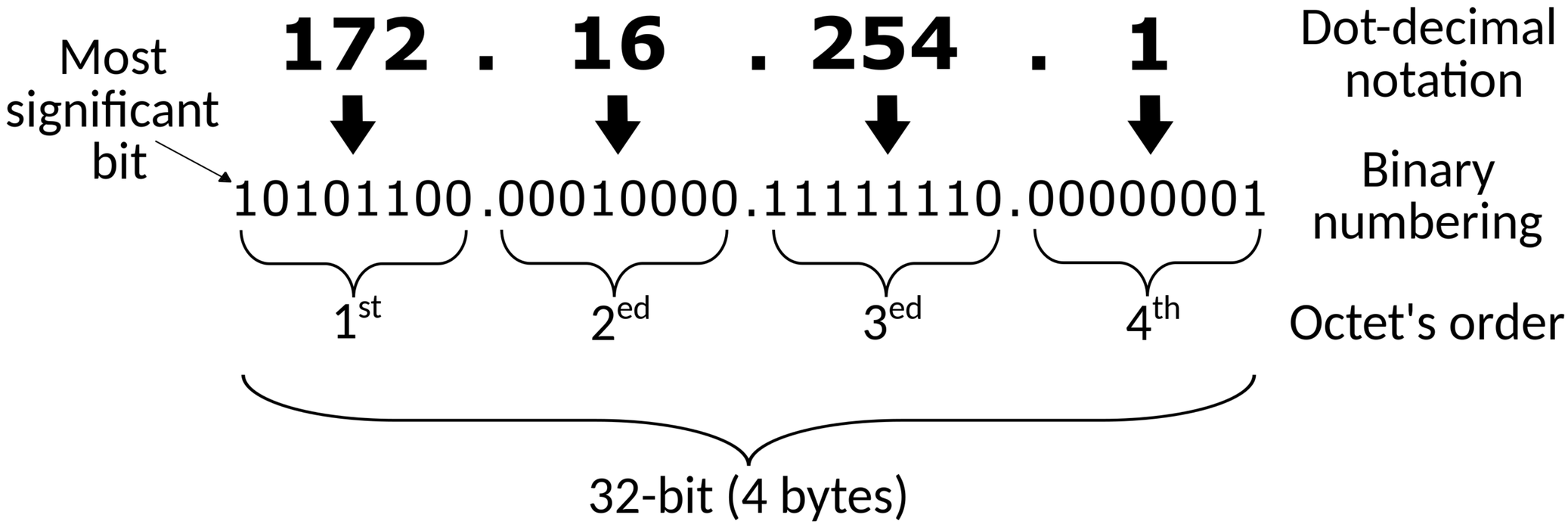
Bit-packing

Example: IP Packet



Bit-packing

Example: IP Address



Bit-packing

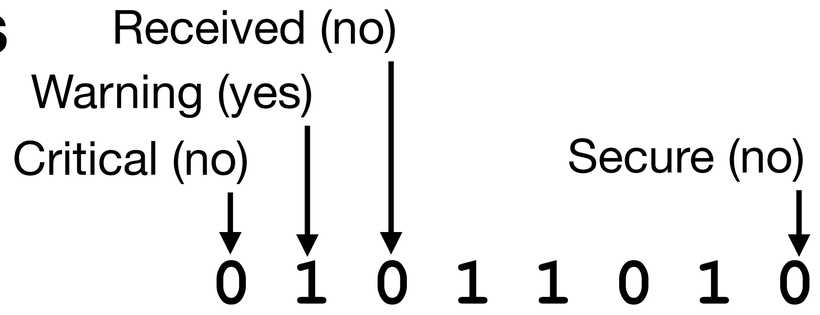
Example: IP Address

IP address	192.168.1.2
Subnet mask	255.255.255.0
Router	192.168.1.1

What is this mask trying to get?

Bit-packing

Example: Flag Bits

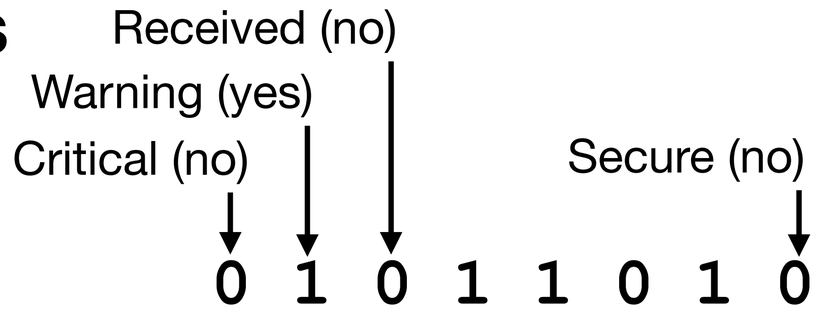


Each bit answers one yes/no question -- 8 Booleans in a byte!

```
const uint8_t CRITICAL = 0x80;   ->10000000
const uint8_t WARNING  = 0x40;   ->01000000
const uint8_t RECEIVED = 0x20;   ->00100000
...
const uint8_t SECURE   = 0x01;   ->00000001
```

Bit-packing

Example: Flag Bits

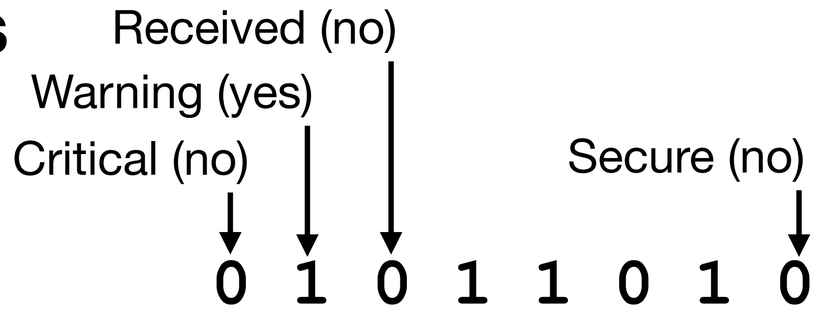



```
const uint8_t CRITICAL = 0x80;   ->10000000
const uint8_t WARNING  = 0x40;   ->01000000
const uint8_t RECEIVED = 0x20;   ->00100000
...
const uint8_t SECURE   = 0x01;   ->00000001

if (flags & CRITICAL) { ... }
```

This is non-zero if and only if the first bit is set.

Bit-packing

Example: Flag Bits



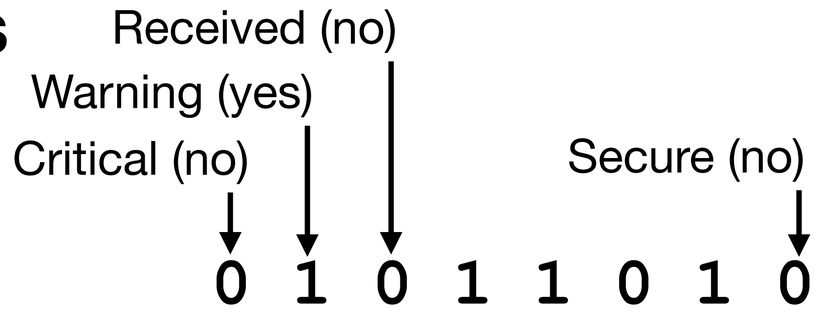

```
const uint8_t CRITICAL = 0x80;   ->10000000
const uint8_t WARNING  = 0x40;   ->01000000
const uint8_t RECEIVED = 0x20;   ->00100000
...
const uint8_t SECURE   = 0x01;   ->00000001

if (flags & (CRITICAL | SECURE)) { ... }
```

10000001

This is non-zero if and only if the first bit and the last bit is set.

Bit-packing

Example: Flag Bits

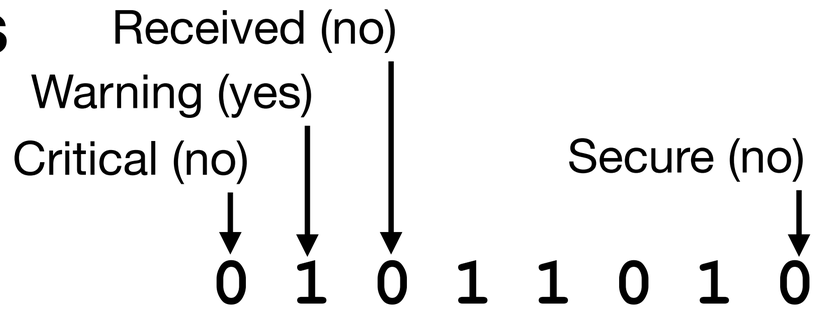



```
const uint8_t CRITICAL = 0x80;   ->10000000
const uint8_t WARNING  = 0x40;   ->01000000
const uint8_t RECEIVED = 0x20;   ->00100000
...
const uint8_t SECURE   = 0x01;   ->00000001


flags |= RECEIVED;
```

This sets the RECEIVED bit.

Bit-packing

Example: Flag Bits

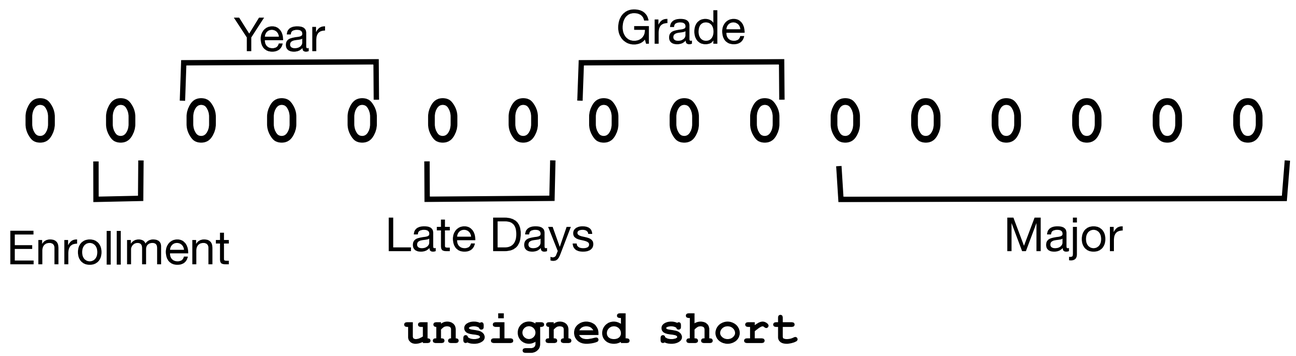



```
const uint8_t CRITICAL = 0x80;   ->10000000
const uint8_t WARNING  = 0x40;   ->01000000
const uint8_t RECEIVED = 0x20;   ->00100000
...
const uint8_t SECURE   = 0x01;   ->00000001


flags &= ~SECURE;
```

This unsets the SECURE bit.

Choices

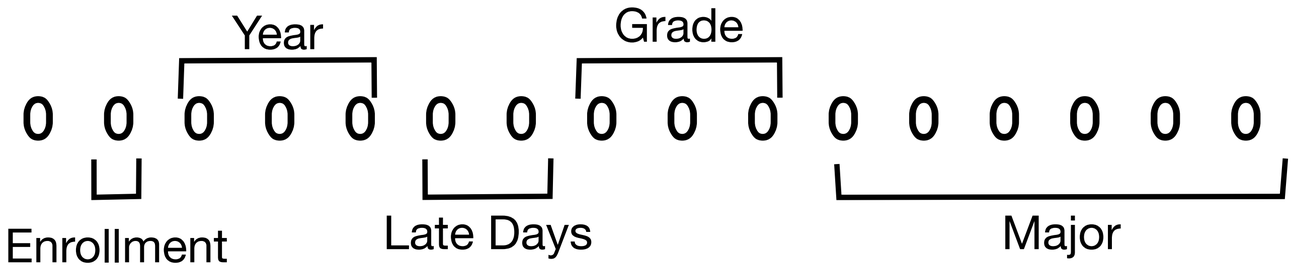
Interlude



- How do we assign bit patterns to majors?
- Anthropology - 0, Architectural Studies - 1, Art History - 2, ...
- One has to look up on a table to figure out which major this is :(

Choices

Interlude: `enum`

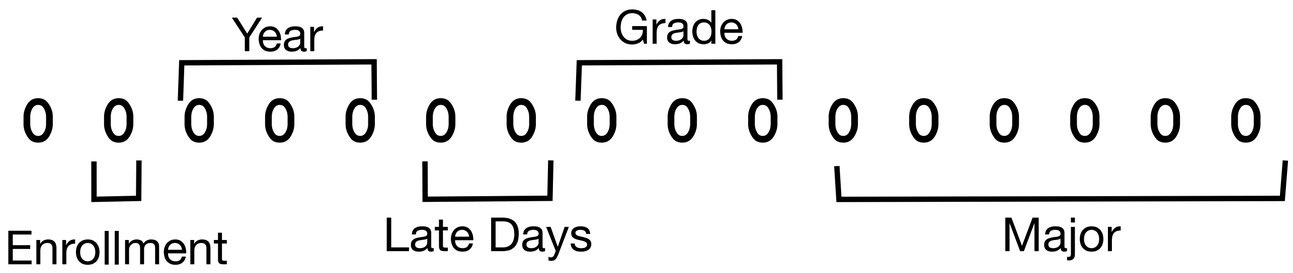



```
enum major {
        ANTHROPOLOGY,
        ARCHITECTURAL_STUDIES,
        ART_HISTORY,
        ASTRONOMY_ASTROPHYSICS,
        BIG_PROBLEMS,
        BIOLOGICAL_CHEMISTRY,
        ...
};
```

- Nothing fancy here: C just assigns an integer sequentially for each choice.
- Can use them as global constants
- `if (major == 2) { ... }`
- `if (major == ART_HISTORY) { ... }`

Choices

Interlude: `enum`



`unsigned short`

```
enum major {
        ANTHROPOLOGY,
        ARCHITECTURAL_STUDIES,
        ART_HISTORY,
        ASTRONOMY_ASTROPHYSICS,
        BIG_PROBLEMS,
        BIOLOGICAL_CHEMISTRY,
        ...
};
```

```
enum major student_major = STATISTICS;
```

```
enum major student_major = 3;
```

`clang` will *not* complaint about this. But this is bad style.

Choices

Interlude: `switch`

```
if (major == ANTHROPOLOGY) {
        ...
} else if (major == ARCHITECTURAL_STUDIES) {
        ...
} else if (major == ART_HISTORY) {
        ...
} else if (major == ASTRONOMY_ASTROPHYSICS) {
        ...
} else if (major == BIG_PROBLEMS) {
        ...
} ...
```

```
switch (major) {
case ANTHROPOLOGY:
        ...;
        break;
case ARCHITECTURAL_STUDIES:
        ...;
        break;
case ART_HISTORY:
        ...;
        break;
case BIOLOGICAL_CHEMISTRY:
        ...;
        break;
case BIG_PROBLEMS:
        ...;
        break;
default:
        break;
}
```

`break` signals the end of a case. Without break, C will execute the next case, *falling through* another case.

The default branch is run when the major matches none of the above cases.

Choices

Interlude: `switch`

- Why switch?
- Cleaner code
- More efficient than if ... else if ... chain:
  - C stores the branches in a table, and switch will jump to the branch instead of comparing one by one
  - `switch(x)`, `x` has to be an integer.
- `break` is critical! Forgetting the break is really difficult to debug.

Choices

Interlude: `switch`

- Demo! if we have time