

# Functional Programming

*Overview: Monoid, Foldable, Traversable*

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foldr (++) "" ["223","00"] :: [Char]

foldr (+) 0 [2,2,3,0,0] :: Int

foldr (\*) 1 [2,2,3,0,0] :: Int

foldr (||) False [True, True, False] :: Bool

foldr (&&) True [True, True, False] :: Bool

foldr firstJust Nothing [Nothing, Just 2, Just 23] :: Maybe Int  
⇒ Just 2

foldr lastJust Nothing [Nothing, Just 2, Just 23] :: Maybe Int  
⇒ Just 23

foldr plusJust Nothing [Nothing, Just 2, Just 23] :: Maybe Int  
⇒ Just 25

foldr multJust Nothing [Nothing, Just 2, Just 23] :: Maybe Int  
⇒ Just 46

foldr	(++)	""	["223", "00"]	::	[Char]
foldr	(+)	0	[2, 2, 3, 0, 0]	::	Int
foldr	(*)	1	[2, 2, 3, 0, 0]	::	Int
foldr	(  )	False	[True, True, False]	::	Bool
foldr	(&&)	True	[True, True, False]	::	Bool
foldr	firstJust	Nothing	[Nothing, Just 2, Just 23]	::	Maybe Int
foldr	lastJust	Nothing	[Nothing, Just 2, Just 23]	::	Maybe Int
foldr	plusJust	Nothing	[Nothing, Just 2, Just 23]	::	Maybe Int
foldr	multJust	Nothing	[Nothing, Just 2, Just 23]	::	Maybe Int

**Semigroup** t => **Monoid** t

- One instance per type, so wrapper types

```
foldr (+) 0 ([2,2,3,0,0]    :: []          Int)
foldr (+) 0 (("CMSC", 223) :: (,) String Int)
foldr (+) 0 ((Right 223)    :: Either a    Int)
```

```

foldr (+) 0 ([2,2,3,0,0]    :: [] Int)
foldr (+) 0 (("CMSC", 223)  :: (,) String Int)
foldr (+) 0 ((Right 223)    :: Either a Int)

```

**Foldable t**

```

foldr    :: (a -> b -> b) -> b -> t a -> b
foldl    :: (b -> a -> b) -> b -> t a -> b
fold     :: Monoid m => t m -> m
foldMap  :: Monoid m => (a -> m) -> t a -> m

elem      :: Eq a      => a -> t a -> Bool
( concat :: t [a] -> [a]

...

```

```

sequenceListOfMaybes :: [Maybe a]      -> Maybe [a]

sequenceListOfMaybes :: [] (Maybe a) -> Maybe ([] a)
sequenceListOfTrees  :: [] (Tree a)  -> Tree ([] a)
sequenceTreeOfMaybes :: Tree (Maybe a) -> Tree (Maybe [] a)

```

t\_outer (t\_inner a)      t\_inner (t\_outer a)

**Traversable t\_outer**

```

sequenceA :: Applicative t_inner =>
    t_outer (t_inner a) -> t_inner (t_outer a)

traverse  :: Applicative t_inner =>
    (a -> t_inner b) -> t_outer a -> t_inner (t_outer a)

```

- traverse is “effectful fmap”
- Traversable simultaneously generalizes Functor and Foldable

