Quantum Operations

Based on Q is for Quantum, Terry Rudolph

Bits and Qubits

All computation occurs on 1's and 0's, stored in a bit (qubit in quantum)

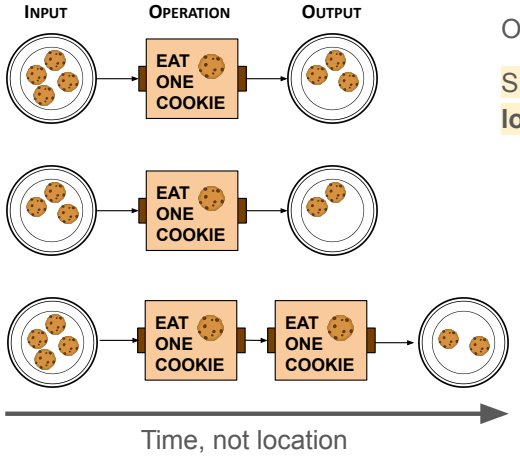
These 1's and 0's cooperate to become

- Numbers (negative, positive, decimal) **98.6**
- Words (each character is a number) HELLO!
- Pictures (each pixel has a number for its color)
- Sounds (each pitch has a number)

Let's learn some bit-level operations!!

Quantum Gates (operations) that act like Classical operations

Imagine we drew operations this way...

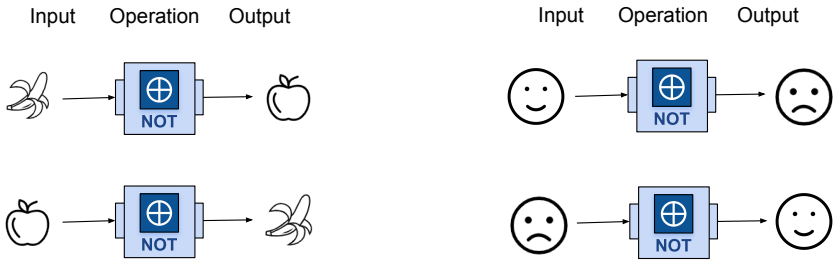


Operations progress from left to right

Shows **order** of operation, **not location** of operation.

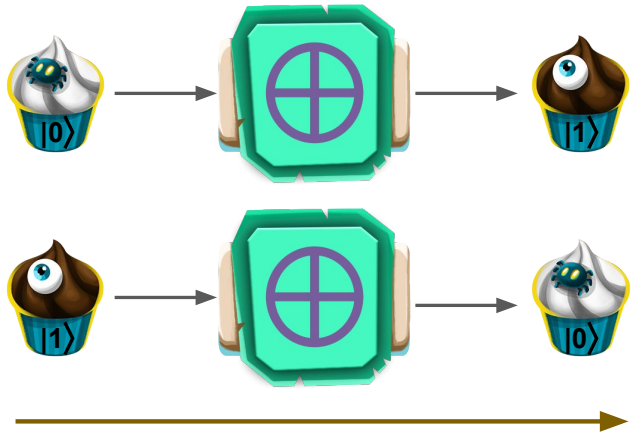
Plate may have never moved

Imagine an operation called NOT



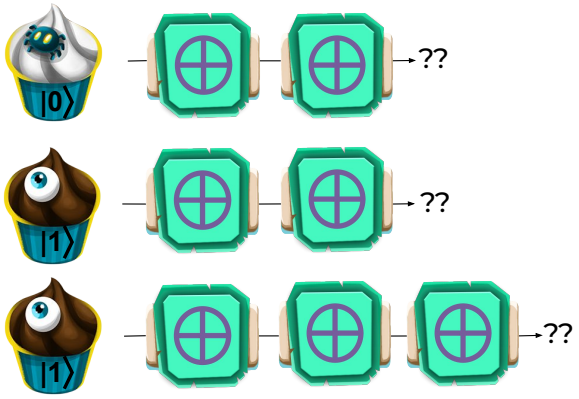
We can use any two items. NOT toggles between them.

New Gate (Operation): NOT or X gate - *flavor / bit flipper*



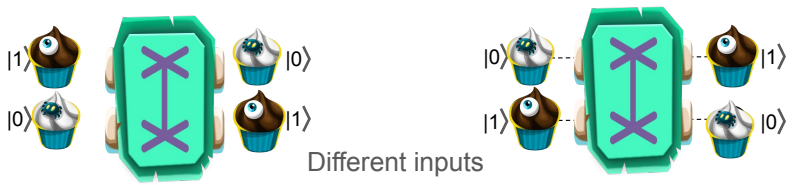
Time goes left to right. Boxes are operations. Qubits might never move!

What would be the outcomes of the following?



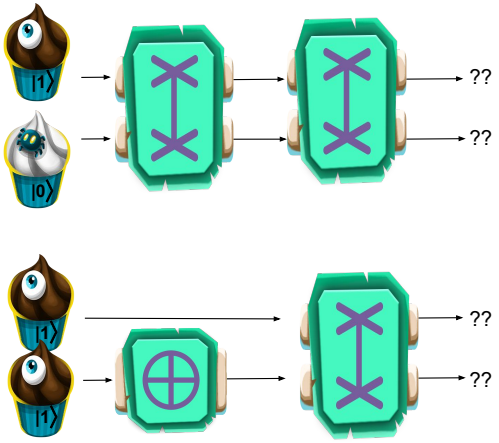
What would be the outcomes of the following?

New Gate: SWAP operation - *flavor swapper*



The cupcakes swap flavors *relative to one other*. Cupcakes **do not move** or swap *locations*. If the cupcakes are the same flavor, *no change occurs*.

What would be the outcomes of the following?



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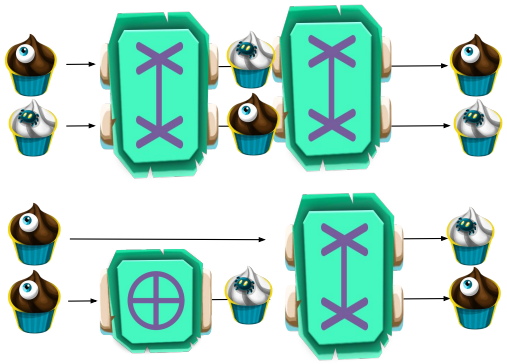
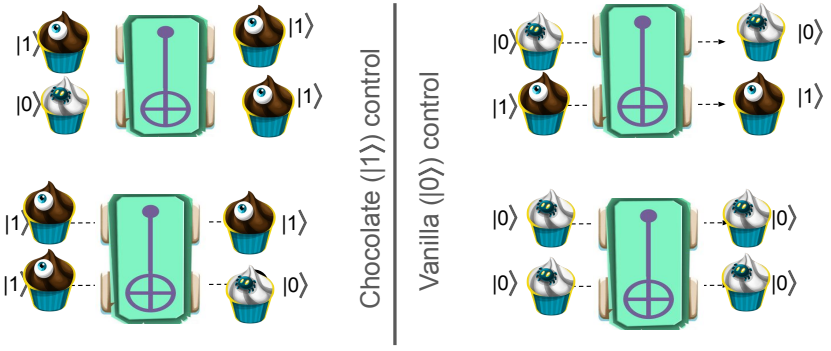
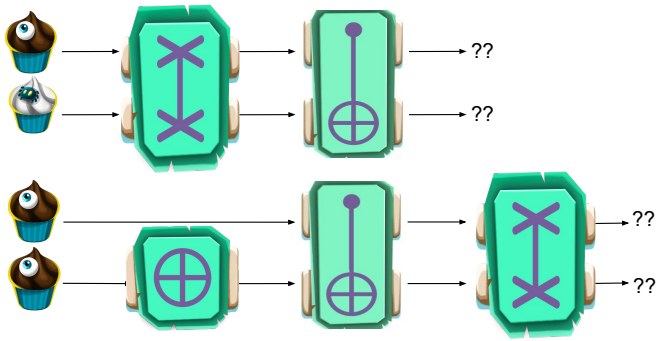


New Gate: CNOT operation - *chocolate-powered NOT*



The **TARGET** flips if and *only* if the **CONTROL** is chocolate (1).

What would be the outcomes of the following?



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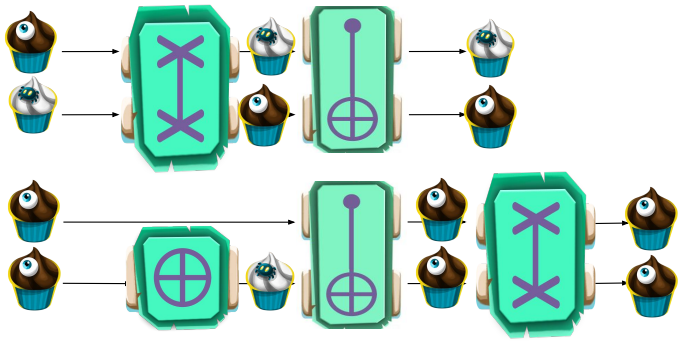

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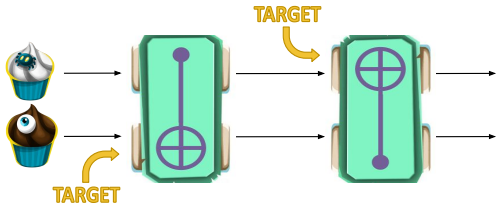

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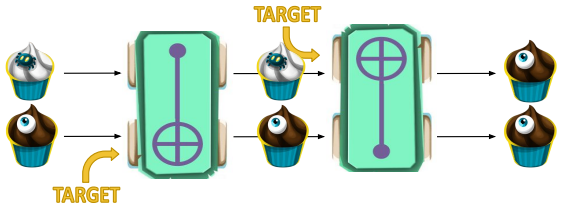


Summary

Quantum computing may accelerate critical applications

Quantum computing will ***assist, not replace*** general-purpose computing

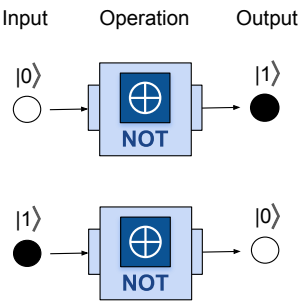
Over the course of the quarter, we will learn quantum-specific operations (that use superposition and entanglement) and how to harness those for useful work!

New Visual Representation (no more cupcakes)

For quantum:

White ball, labeled |0⟩

Black ball, labeled |1⟩



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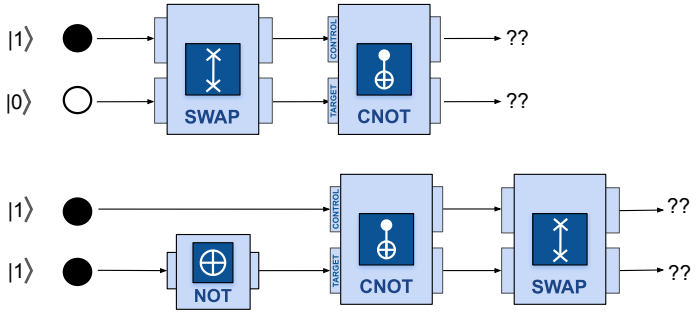

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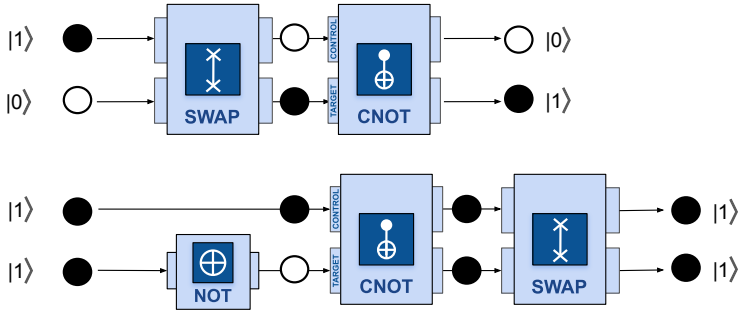


Quantum Concepts and the Gates they Influence

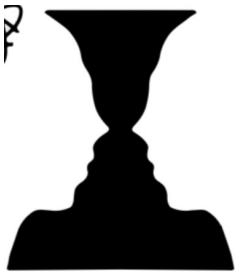
What does this mean for computing?

The qubit is in **superposition of both values at the same time.**

Superposition: Two things at once

Rabbit or Duck?

Vase or Faces?



Superposition: Holds multiple values at once

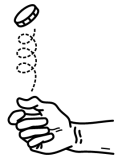
Superposition:
Holds 2 values at once

A bit: Stores either the value 0 or 1 at any given time

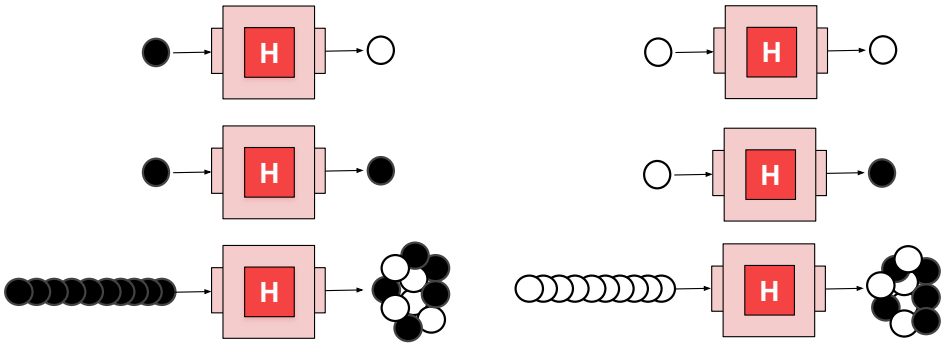
X = 5;

A qubit: Stores several things, including both 0 and 1

X = 5 and 8 and 12;



New Gate: The **H** Gate



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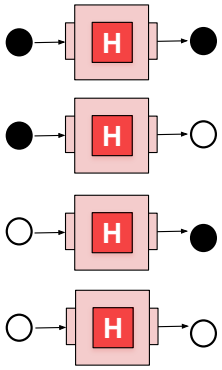
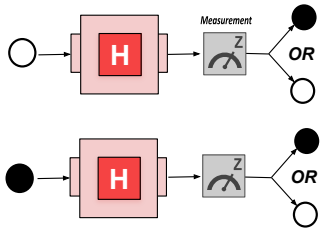
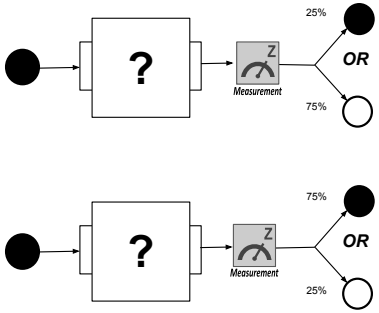


Observed outcome from single gate is random!



Probability (50%), not outcome, is predictable.

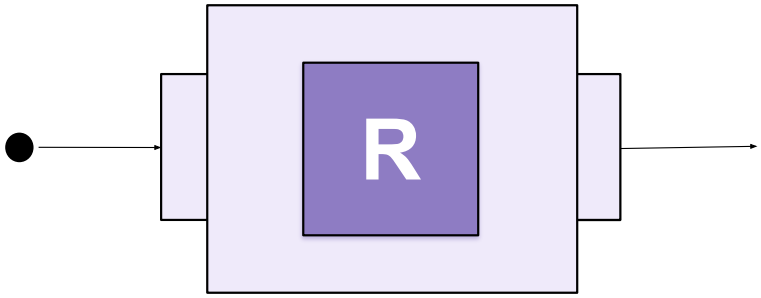
Gates exist that result in the following



Learning Goals

- ➔ What is going on with these probabilistic gates?
- ➔ How do we represent the probabilistic output visually?
- ➔ How do we represent the probabilistic output mathematically?
- ➔ How can we calculate these probabilities as they pass through other gates?

Does the H gate really cause superposition?
Or is the H gate a *random* gate producing random outputs?



Random Thoughts...

*Or thoughts about the word **random***

That was such a ***random*** comment!

That was ***unpredictable*** - to me!

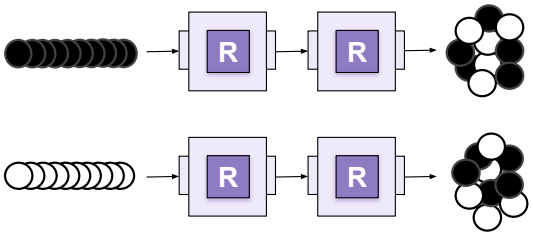
Draw a number at ***random***!

Each number has ***equal probability*** of being drawn.

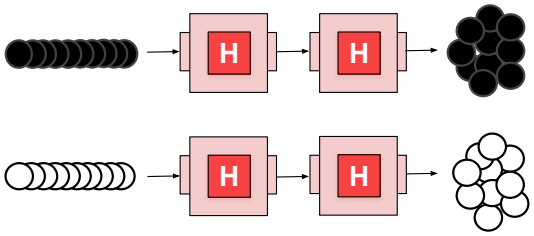
Quantum measurements have ***random*** outcomes!

We know the ***probability*** of an outcome, but the outcome of a single measurement is ***not guaranteed*** (indeterminate or nondeterministic)

If it is an internal coin flip, what is the expected outcome?

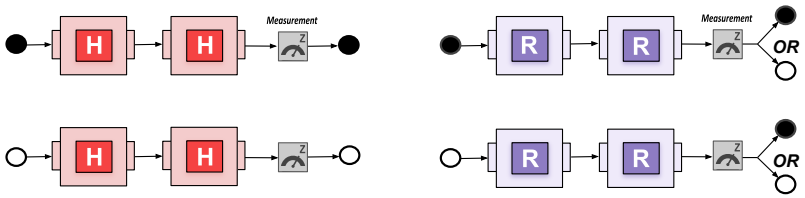


Here is what actually happens...



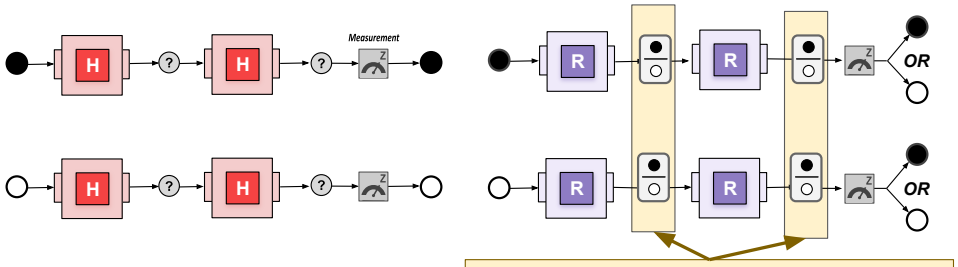
Deduction: **H** gate ***appears*** random, but it is not!

What makes the **H** *gate different from a purely random gate* (**R**)*?*



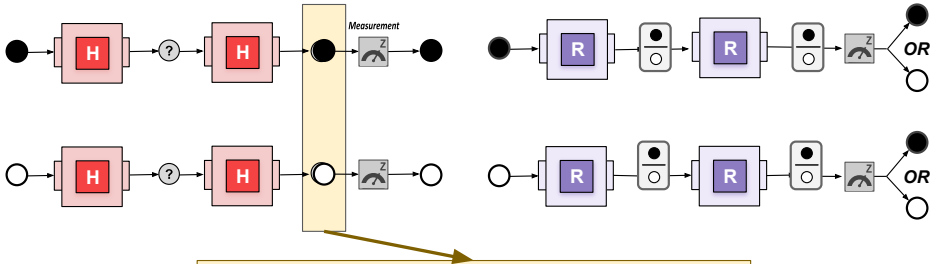
Two instances ***always*** brings it back to the initial color.
Therefore, it cannot be random.

Representing the 50/50 superposition state



A random gate results in an output that is 50/50 black vs white, which we depict has the two possibilities in this way.

Deductions:

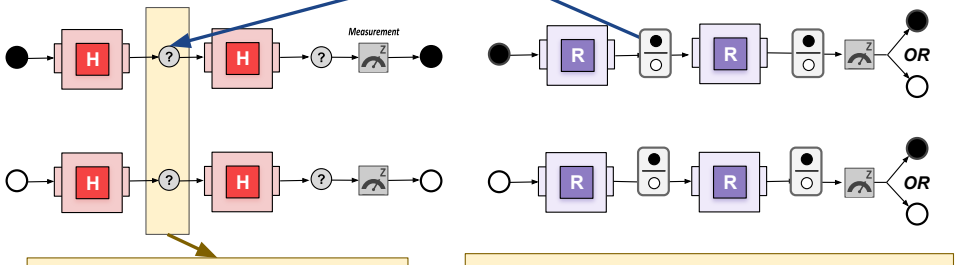


Output from initial black input: Always black

Output from initial white input: Always white

Deductions:

Why not just measure the exact states and see how they are different?!?



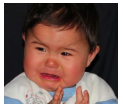
A black ball's output is different from a white ball's output.

After an **H** gate, but before measurement, **there is some aspect of state we have not captured**

Heisenberg Uncertainty Principle: You Can't Accurately **Measure** / Read out the Complete State

Superposition:
Holds 2 values at once
Cannot read full state

Happy
Proud
Excited



Sad
Hungry
Tired
Angry
Bored
Anxious

Heisenberg Uncertainty Principle: You Can't Accurately **Measure** / Read out the Complete State

Superposition:
Holds 2 values at once
Cannot read full state

A bit: Stores either the value 0 or 1 at any given time

```
Int x = 5;
println("X = " + X);

>> java cstate
>> X = 5
>> java cstate
>> X = 5
>> java cstate
>> X = 5
```

A qubit: Stores several things, including both 0 and 1

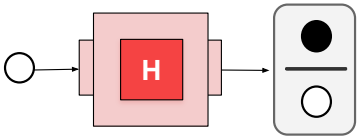
```
X = 5 and 8 and 12;
println("X = " + X);

>> java cstate
>> X = 5
>> java cstate
>> X = 12
>> java cstate
>> X = 5
```

X has three values, but we only read one of them

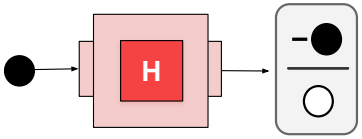
Flu or COVID?

Mathematically, it means we were missing part of the state

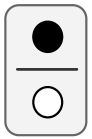
Equal probability of each **0 / 1** outcome.

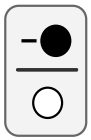


The negative sign (-) indicates an additional aspect of state.



Phase

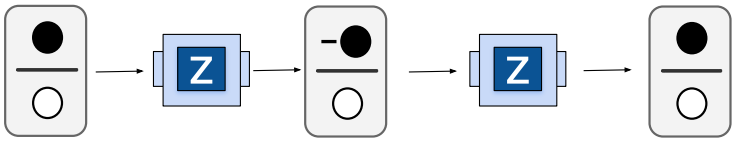




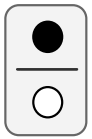
- Positive vs Negative phase
- *Phase can also be in superposition*!! If we were to measure phase, not 0/1,
  - Positive phase: > 50% of measuring +
  - Negative phase: < 50% of measuring +

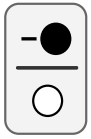
Phase Flip Gate: Z

- Like a NOT gate, but flips probability of measuring positive vs negative phase



Phase: Looking ahead

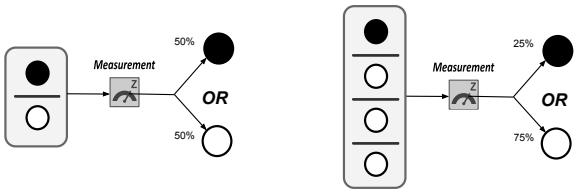




- How to calculate operations with negative phase inputs
- Surprising behavior of phase
- How to use phase in a quantum algorithm
- How to calculate exact probability of measuring + vs -

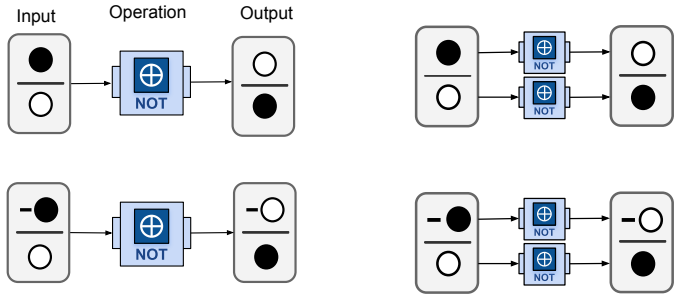
Visual Superposition State

Beyond 50/50 Superposition



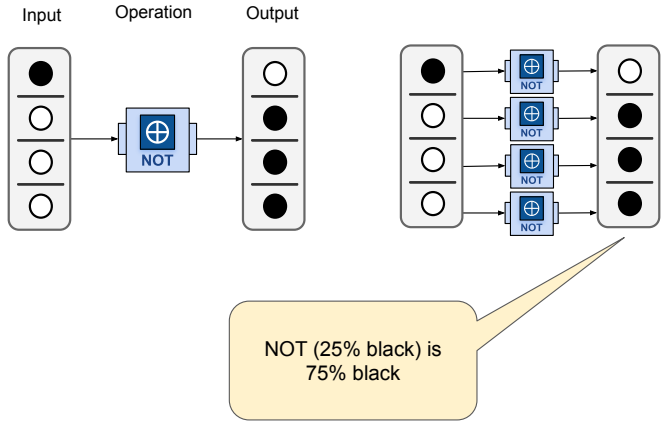
Each horizontal line distinguishes between equally likely measurement outcomes

Superposition as input:

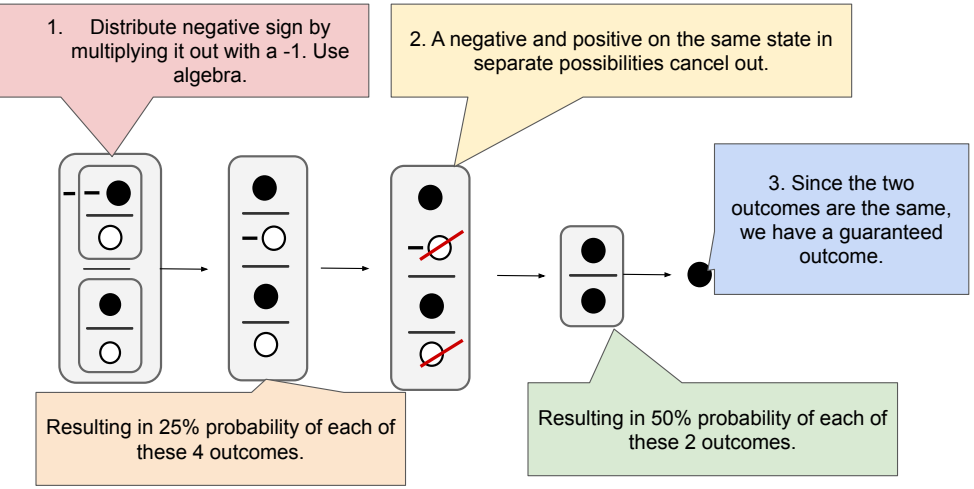


1. To apply a gate, apply separately to each possibility

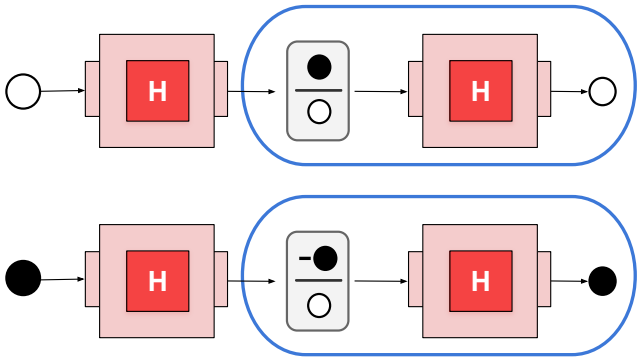
Superposition as input:



Visual Representation Simplification Rules



Revisiting odd H-gate behavior...



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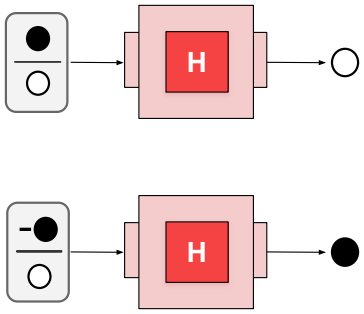
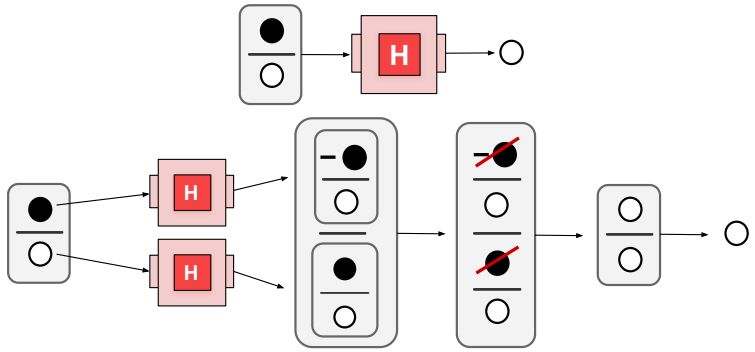


Revisiting odd behavior...



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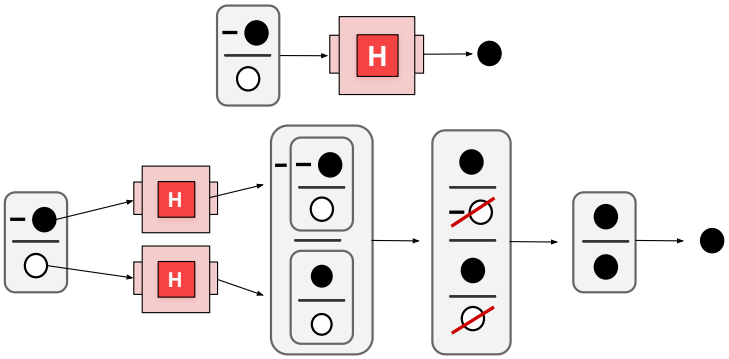
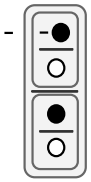
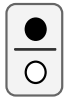
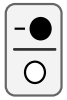


Select the option(s) that describes the same quantum state as:



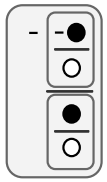
A. 

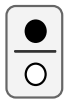
B. 

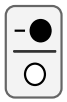
C. ○

D. ●

Select the option(s) that describes the same quantum state as:



A. 

B. 

C. ◯

D. ●

Summary

- ➔ The **H** gate puts a qubit in superposition.
- ➔ An **H** gate applied to a state of |0⟩ or |1⟩ results in 50/50 chance of measuring 0 or 1.
- ➔ Two **H** gates in sequence reverse each other, resulting in the original input.
- ➔ Thus, there is more to state than just the probability of measuring 0 or 1 - there is also ***phase***.
- ➔ Our calculation with the phase value accurately models / predicts this reversing behavior.

Homework Reminder

- Programming project on 22880 website: https://www.classes.cs.uchicago.edu/archive/2022/winter/22880-1/assigns/index.html
- Short-answer questions on Gradescope
- Questions on Ed Discussion

Randomness and Independent Probabilities

What does the probability of rain really mean?!?

What can we conclude from this weather forecast?

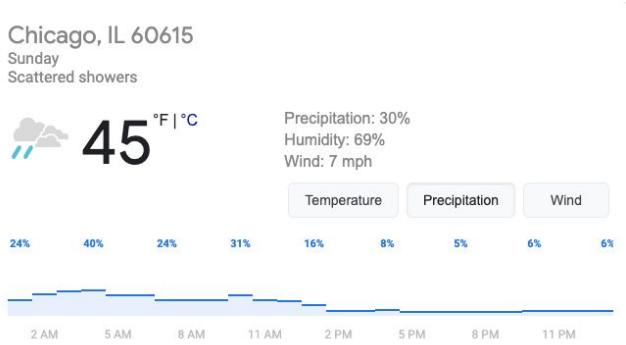
It is not going to rain at 11pm?
Not a valid conclusion

It is going to rain at 4am?

Not a valid conclusion

It is more likely to rain at 8am than 8pm?

Valid conclusion

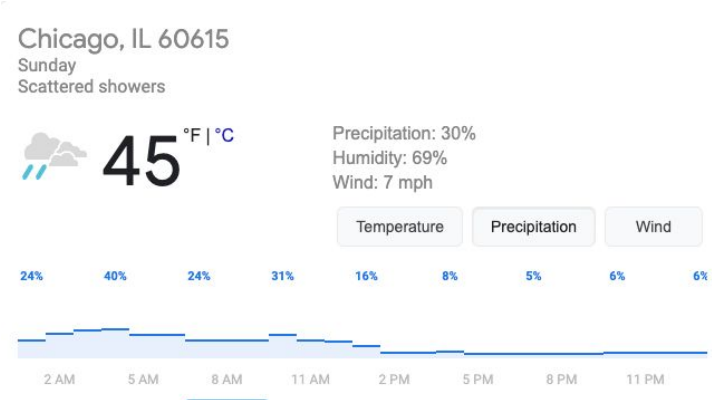


It does not tell us whether or not it will rain on Sunday.

What does the probability of rain really mean?!?

Given our limited information:

If we experienced 100,000 days with identical conditions, approximately 40,000 of them would have rain at 4am.



Probability is **not**….

A prediction about the outcome of any individual action

Confirmed or refuted through a single experiment

Probability is….

A prediction of the frequency of an outcome of many, many actions

Confirmed or refuted through many, many experiments

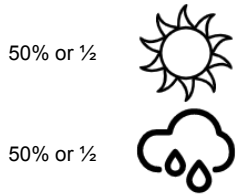
Combining multiple independent events

There is a 50% chance (probability) of rain in the afternoon.

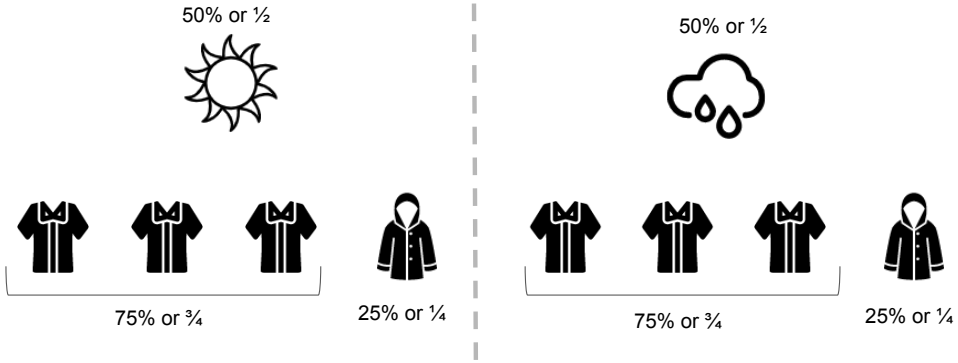
There is a 25% chance (probability) I will remember my raincoat.

What is the probability that it will both rain **and** I will forget my raincoat?

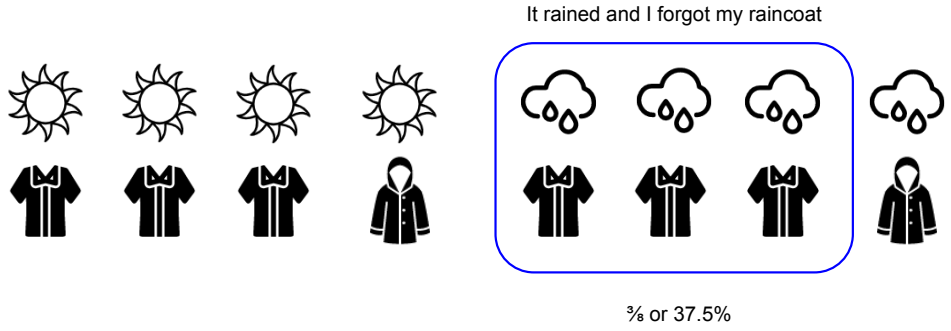
Visual representation depicts the independent probabilities



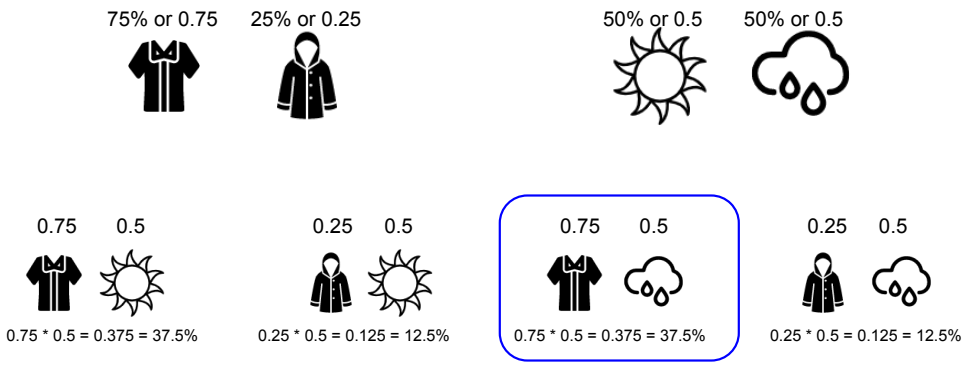
To combine them, we can make combinations



8 equally-likely, non-unique scenarios



Or…. calculate to get each unique outcome



How is probability used in Quantum Computing?

Each qubit in superposition has a probability of being measured 0 or 1.

Multiple qubits are required in order to perform useful computation.

Qubits start with independent probabilities, but then they become multi-qubit combinations.

Random Thoughts...

*Or thoughts about the word **random***

That was such a ***random*** comment!

That was ***unpredictable*** - to me!

Draw a number at ***random***!

Each number has ***equal probability*** of being drawn.

Quantum measurements have ***random*** outcomes!

We know the ***probability*** of an outcome, but the outcome of a single measurement is ***not guaranteed*** (indeterminate or nondeterministic)

Superposition & Measurement

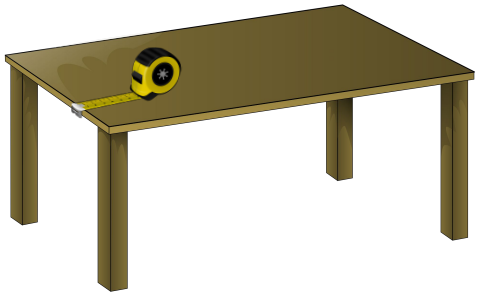
Measurement

Consists of a ***question***, a ***device,*** and a ***method***

Question: What is the table length?

Device: Tape Measure

Method:
Pull out the tape measure along the length of the table and read out the number at the end.



Measurement

Consists of a ***question***, a ***device*** and a ***method***

Question: What is this baby feeling?

Device: Eyes and ears

Method:
Look for smile, frown, or tears.
Listen for laughs, silence, or screams.

- Happy
- Excited
- Proud

- Hungry
- Frustrated
- Tired

Some measurements give only partial information!

Measurement

Consists of a ***question***, a ***device*** and a ***method***

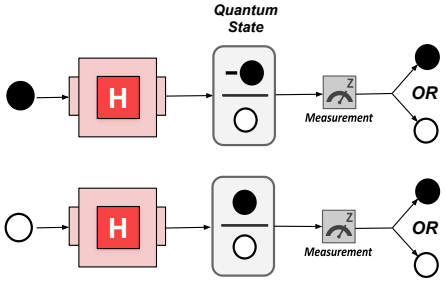
Question: How long can you hold your breath?

Device: Stopwatch

Method:
Use a stopwatch to time how long you can hold your breath.

Some measurements affect the item being measured!

Quantum Measurement



Measurement does not reveal full state!
Measurement reveals neither phase nor probabilities involved!

Superposition

- A single object can be multiple things at once
- State is suspended as a combination of multiple values

Measurement resolves a superposition

Question: Which definition of **polish** is this?

Need a **device** and **method**.

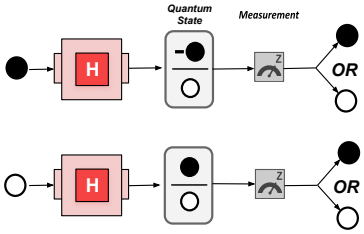
Your **ear** **hears** the word said aloud.

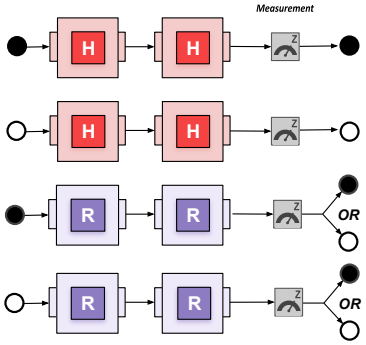
- "**Paw**-lish" \ ˈ**pä**-lish \
- "**Poe**-lish" \ ˈ**pō**-lish \

Your **eyes** **read** the words around it for context.

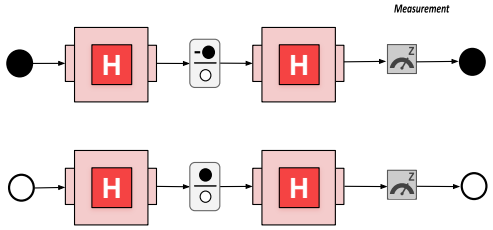
- "**Polish** makes the floor shine."
- "**Polish** sausage is delicious."

Revisiting H Gate: The Role of Measurement



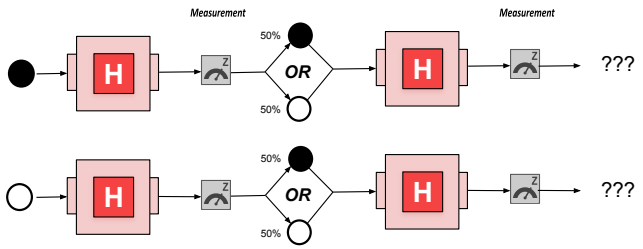


Predictable outcome w/out intermediate measurement

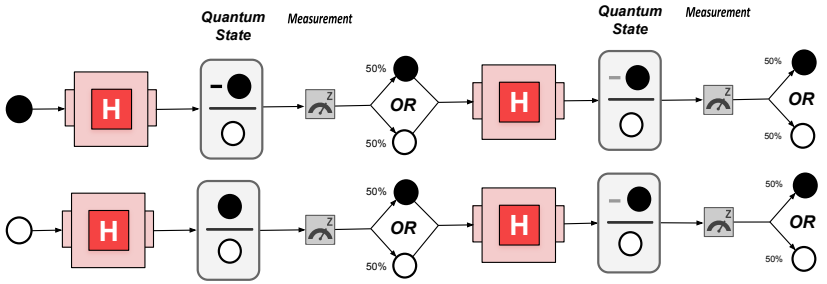


Superposition of complex state enters second gate

What happens if we try to observe the balls in between...



Unpredictable outcome with intermediate measurement



Measurement collapses superposition

In Quantum, measurement collapses superposition

Superposition:
Holds 2 values at once
Cannot read full state

A bit: Stores either the value 0 or 1 at any given time

```
Int x = 5;
println("X = " + X);
println("X = " + X);
>> java cstate
>> X = 5
>> X = 5
>> java cstate
>> X = 5
>> X = 5
```

A qubit: Stores several things, including both 0 and 1

```
X = 5 and 8 and 12;
println("X = " + X);
println("X = " + X);
>> java cstate
>> X = 5
>> X = 5
>> java cstate
>> X = 12
>> X = 12
```

Once read, it no longer holds all 3 and will get the same result without resetting the program.

Quantum Superposition

- ➔ A qubit is a superposition of two values: $|0\rangle$ or $|1\rangle$
- ➔ Part of quantum state is the ***probability*** of measuring 0 or 1
- ➔ The ***probability*** that a measurement detects one or the other can be ***manipulated*** through quantum operations
- ➔ ***Measurement*** cannot detect the entire state, only an individual 0 or 1
- ➔ The act of ***measurement collapses the superposition***, making the qubit become only the measured value 0 or 1