CMSC 23700	**Introduction to Computer Graphics**	**Homework 1**
Autumn 2017		**Due October 5**

1. Prove that $\sqrt{\mathbf{u} \cdot \mathbf{u}} = \|\mathbf{u}\|$ for any vector $\mathbf{u}$ using the definition of dot product.

2. Let $\mathcal{B} = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$ be an orthonormal basis and let $\mathbf{B} = [\mathbf{b}_i]$ be the matrix whose columns are the basis vectors.
   Prove that $\mathbf{B}^{-1} = \mathbf{B}^{\mathrm{T}}$.

3. Prove that for any two non-zero vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$,
   $$(\mathbf{u} \cdot \mathbf{v})^2 + \|\mathbf{u} \times \mathbf{v}\|^2 = \mathbf{u}^2\mathbf{v}^2$$
   (where the notation $\mathbf{u}^2$ means $\mathbf{u} \cdot \mathbf{u}$).

4. Prove that for any three vectors $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$,
   $$(\mathbf{u} \times \mathbf{v}) \times \mathbf{w} = (\mathbf{u} \cdot \mathbf{w})\mathbf{v} - (\mathbf{v} \cdot \mathbf{w})\mathbf{u}$$

5. Consider two unit vectors, $\mathbf{u}$ and $\mathbf{v}$. The *linear interpolation* between these vectors is defined to be
   $$\mathrm{lerp}(\mathbf{u}, t, \mathbf{v}) = (1 - t)\mathbf{u} + t\mathbf{v}$$
   where $0 \leq t \leq 1$. While this operation works well when the vectors represent positions, it does not work well when the vectors represent directions, since the angle between $\mathbf{u}$ and $\mathrm{lerp}(\mathbf{u}, t, \mathbf{v})$ is not a linear function of t.

   Give psuedocode for a function $\mathit{slerp}(\mathbf{u}, t, \mathbf{v})$, where $0 \leq t \leq 1$, that returns a unit vector $\mathbf{w}$, such that the angle between $\mathbf{u}$ and $\mathbf{w}$ is a linear function of t.