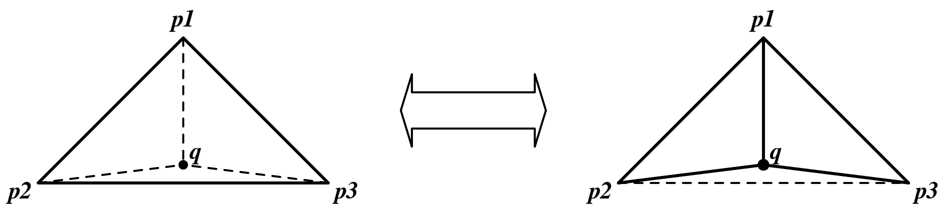
CMSC 23700	**Introduction to Computer Graphics**	**Homework 4**
Autumn 2017		**Due November 16**

1. One way to make level-of-detail (LOD) transitions is to use an α *fade*, where you lerp the α channel to blend the two LODs. For example, assume that you have a triangle $\triangle\langle \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\rangle$ and a vertex $\mathbf{q}$ that splits the triangle into two triangles $\triangle\langle \mathbf{p}_1, \mathbf{p}_2, \mathbf{q}\rangle$ and $\triangle\langle \mathbf{p}_1, \mathbf{q}, \mathbf{p}_3\rangle$ as follows:



   At the coarse LOD (the left side), we just render $\triangle\langle \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\rangle$, and at the fine LOD (the right side), we render both $\triangle\langle \mathbf{p}_1, \mathbf{p}_2, \mathbf{q}\rangle$ and $\triangle\langle \mathbf{p}_1, \mathbf{q}, \mathbf{p}_3\rangle$, but in between we render all three triangles and use alpha blending to smooth the transition.

   Assuming that $0 \leq t \leq 1$, give the blending equation that describes how to combine the two images as a function of t. It should be the case that when $t = 0$, just the coarse LOD is rendered and when $t = 1$, just the fine LOD is rendered.

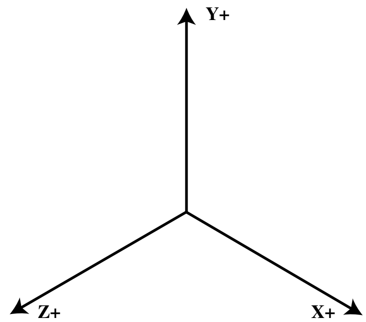
2. Consider a sphere tree with the following representation:

```
struct SphereTree {
    vec3f       center;     // center of the sphere
    float       radius;     // radius
    float       radius2;    // radius*radius
    SphereTree *kid;        // children, which are contained in this
    SphereTree *kid;        // sphere (or nullptr if this is a leaf)
};
```

   (a) Sketch a function that given two sphere trees, `st1` and `st2`, creates a new sphere tree node with **minimum** radius that has `st1` and `st2` as its children.

   (b) Sketch an **efficient** function that when given a ray and a sphere tree, returns true if a ray intersects any of the leaves of the tree. Note that you will first need to define a function that tests ray-sphere intersection.

3. An *isometric projection* is a parallel projection in which the angles between the projected axes are equal (*i.e.*, $120°$) as shown in the following picture.



Let f be the distance to the far plane and n the distance to the near plane. Assume that the sides of the view volume are given by $r = 1$, $l = -1$, $t = 1$, and $b = -1$. Define an isometric projection matrix that maps the world-space axes as shown in the picture, with the world-space origin being projected to $x = 0$ and $y = 0$.