CMSC 28100-1 / MATH 28100-1
Introduction to Complexity Theory
Fall 2017 – Homework 3

October 12, 2017

Exercise 1 (HMU 8.2.2)**.** Design Turing machines for the following languages.

(a) The set of strings with an equal number of 0's and 1's.

(b) $\{a^n b^n c^n : n \geqslant 1\}$.

(c) $\{ww^R : w \text{ is any string of 0's and 1's}\}$, where w^R is the reverse of a string. For instance, we have $10010^R = 01001$.

Exercise 2 (HMU 8.2.3)**.** Design a Turing machine that takes as input a number N in binary and adds 1 to it. To be precise, the tape initially contains a \$ followed by N in binary. The head is initially scanning the \$ in state q_0. Your Turing machine should halt with $N+1$ in binary on its tape, scanning the leftmost symbol of $N+1$, in state q_f. You may destroy the \$ in creating $N+1$, if necessary. For instance, $q_0\$10011 \mapsto \$q_f10100$ and $q_0\$11111 \mapsto q_f100000$.

(a) Give the transitions of your Turing machine, and explain the purpose of each state.

(b) Show the sequence of instantaneous descriptions (IDs) of your Turing machine when given input \$111.

Exercise 3 (HMU 8.2.5)**.** Recall that the language $L(M)$ defined by a Turing machine M is the set of input strings x such that M on input x halts in an accepting state. (M halts when there is no entry in the transition function for the current state symbol pair.)

Consider the Turing machine M with states $\{q_0, q_1, q_2, q_f\}$, input alphabet $\{0, 1\}$, tape alphabet $\{0, 1, B\}$, start state q_0, accepting state q_f, and transition function δ. For each of the following transition functions δ, *informally but clearly describe* the language $L(M)$ if δ consists of the following set of rules.

(a) δ is given by

$$\delta(q_0, 0) = (q_1, 1, R); \qquad \delta(q_1, 1) = (q_0, 0, R); \qquad \delta(q_1, B) = (q_f, B, R).$$

(b) δ is given by

$$\delta(q_0, 0) = (q_0, B, R); \qquad \delta(q_0, 1) = (q_1, B, R);$$
$$\delta(q_1, 1) = (q_1, B, R); \qquad \delta(q_1, B) = (q_f, B, R).$$

(c) δ is given by

$$\delta(q_0, 0) = (q_1, 1, R); \qquad \delta(q_1, 1) = (q_2, 0, L);$$
$$\delta(q_2, 1) = (q_0, 1, R); \qquad \delta(q_1, B) = (q_f, B, R).$$

Exercise 4 (HMU 8.4.3)**.** *Informally but clearly describe* nondeterministic Turing machines (multiple tape, if you like) that accept the following languages. Try to take advantage of nondeterminism to avoid iteration and save time in the nondeterministic sense. That is, prefer to have your nondeterministic Turing machine nondeterministically branch a lot, while each branch is short.

(a) The language of all strings of 0's and 1's that have some string of length 100 that repeats, not necessarily consecutively. Formally, this language is the set of strings of 0's and 1's of the form $wxyxz$, where $|x| = 100$, and w, y and z are strings of arbitrary length (possibly zero).

(b) The language of all strings of the form $w_1\#w_2\#\cdots\#w_n$ for any n such that each w_i is a string of 0's and 1's, and for some j, w_j is the integer j in binary.

(c) The language of all strings of the same form as (b), but for at least two values of j, we have w_j equal to j in binary.