CMSC 28100-1 / MATH 28100-1
Introduction to Complexity Theory
Fall 2017 – Bonus Homework

November 16, 2017

Exercise 1 (HMU 8.4.1)**.** Informally but clearly describe multitape Turing machines that accept each of the following languages from HW3, Exercise 1. Try to make your Turing machines run in time $O(n)$.

(a) The set of strings with an equal number of 0's and 1's.

(b) $\{a^n b^n c^n : n \geqslant 1\}$.

(c) $\{ww^R : w$ is any string of 0's and 1's$\}$, where w^R is the reverse of a string. For instance, we have $10010^R = 01001$.

Exercise 2. Design (single tape) Turing machines for the following languages.

(a) $L = \{w \in \{0,1\}^* : w$ contains more zeroes than ones$\}$.

(b) $L = \{w \in \{0,1\}^* : w$ contains exactly twice as many zeroes as ones$\}$.

Exercise 3 (HMU 8.3.2)**.** A common operation in Turing machine programs involves "shifting over": ideally, we would like to create an extra cell at the current head position, in which we could store some character. However, we cannot edit the tape in this way. Rather, we need to move the contents of each of the cells to the right of the current head position one cell right, and then go our way back to the current head position. Show how to perform this operation. Hint: leave a special symbol to mark the position to which the head must return.

Exercise 4. Give a sufficiently detailed description of how to construct a (multitape) Turing machine which computes the exponentiation function $f(n) = 2^n$. You can assume the input n is given in unary form 1^n, and your Turing machine should produce a unary representation of the number 2^n (i.e., your machine has to produce 1^{2^n}), and halt.